Supersymmetric reduced minimal 3-3-1 model

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Abstract

We build an supersymmetric version of the minimal 3-3-1 model with just two Higgs triplets using the superfield formalism. We study the mass spectrum of all particles in concordance with the experimental bounds. At the tree level, the masses of charged gauge bosons are the same as those of charged Higgs bosons. We also show that the electron, muon and their neutrinos as well as down and strange quarks gain mass through the loop correction. The narrow constraint on the ratio $t_w = \frac{w}{w'}$ is given by studying the new invisible decay mode of the Z boson.

Key words: Supersymmetric models, Extensions of electroweak Higgs sector, Supersymmetric partners of known particles
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1 Introduction

Models with $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ gauge symmetry (called 3-3-1 models for short) are interesting possibilities for the physics at the TeV scale [1,2,3,4]. The 3-3-1 models can have several representation contents depending on the embedding of the charge operator in the $\mathrm{SU}(3)_L$ generators,

$$\frac{Q}{e} = \frac{1}{2}(\lambda_3 - \vartheta\lambda_8) + X \;\; I, \tag{1}$$

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where the ϑ parameter defines the different representation contents, X denotes the $\mathrm{U}(1)_X$ charge and λ_3, λ_8 are the diagonal generators of SU(3).

In fact, this may be the last symmetry involving the lightest elementary particles: leptons. The lepton sector is exactly the same as in the Standard Model (SM) [5] but now there is a symmetry, at large energies among, say e^-, ν_e and e^+. Once this symmetry is imposed on the lightest generation and extended to the other leptonic generations it follows that the quark sector must be enlarged by considering exotic charged quarks. It means that some gauge bosons carry lepton and baryon quantum numbers. Although these models coincide at low energies with the SM it explains some fundamental questions that are accommodated, but not explained in the SM, namely

(1) The family number must be three;
(2) It explains why $\sin^2\theta_W < \frac{1}{4}$ is observed;
(3) They are the simplest models that include bileptons of both types: scalar and vectors ones;
(4) It solves the strong CP problem, the Peccei-Quinn symmetry occurs also naturally in these models [6];
(5) The models have several sources of CP violation [7,8];
(6) Allow the quantization of electric charge [9];
(7) Since one generation of quarks is treated differently from the others this may lead to a natural explanation for the large mass of the top quark [12];
(8) The models also produce a good candidate for Self Interacting Dark Matter (SIDM) since there are two Higgs bosons, one scalar and one pseudoscalar, which have the properties of candidates for dark matter like stability, neutrality and that it must not overpopulate the universe [13], etc.

Another interesting thing about this kind of models is that the gauge 3-3-1 symmetry is considered a subgroup of the popular E_6 Grand Unified Theory (GUT), which can be itself derived from $E_8 \otimes E_8$ heterotic string theory [10,11].

In the minimal version, with $\vartheta = \sqrt{3}$, the charge conjugation of the right-handed charged lepton for each generation is combined with the usual $\mathrm{SU}(2)_L$ doublet of left-handed leptons components to form an SU(3) triplet $(\nu, l, l^c)_L$ [2]. No extra lepton is needed in the mentioned model, and we shall call such model as minimal 3-3-1 model. There are also another possibility where the triplets $(\nu, l, L^c)_L$ contain the extra charged leptons (L). The new charged leptons (L) do not mix with the known leptons [3]. We would like to remind that there is no right-handed (RH) neutrino in both models. There exists another interesting possibility ($\vartheta = 1/\sqrt{3}$), where a left-handed anti-neutrino to each usual $\mathrm{SU}(2)_L$ doublet is added to form an SU(3) triplet $(\nu, l, \nu^c)_L$ [4], and this model is called the 3-3-1 model with RH neutrinos. The 3-3-1 models have been studied extensively over the last decade, see for example

[14,15,16,17,18,19,20].

Despite attractive properties mentioned above, the usual 3-3-1 models have the weakness that reduces their predictive possibility is a plenty in the scalar sectors. The attempt to realize simpler scalar sectors has recently been constructed 3-3-1 model with minimal Higgs sector called the economical 3-3-1 model [21,22]. The 3-3-1 model with minimal content of fermions and Higgs sector (called the reduced minimal (RM) 3-3-1 model) has also been constructed in [23].

The supersymmetric version of the minimal 3-3-1 model [2] has been constructed in Refs. [11,24,25,26] (MSUSY331) while the version with RH neutrinos [4] has already been constructed in Ref. [27,28,29] (SUSY 331RN). The supersymmetric economical 3-3-1 model has been presented recently [30] (SUSYECO331). Some others interesting supersymmetric extensions of the 3-3-1 models were presented in Ref. [31,32,33,34,35].

In this article we will present an supersymmetric version of the reduced minimal 3-3-1 model with the triplet $(\nu, l, l^c)_L$ using only two triplets in the scalar sector.

The outline of the paper is as follows. In section 2 we present representations of fermions and Higgs bosons contained in the supersymmetric RM 3-3-1 model. The super-Lagrangian in terms of superfields is studied in section.(3). In sections 4,5,6, we present the mass eigenstates of gauge bosons, fermions and Higgs bosons as well as the phenomenological consequence of the model under consideration. The Lagrangians in term of fields are given in the Appendix A. In the last section 7, we summary our results and given conclusions.

2 The supersymmetric RM 3-3-1 model

In order to consider supersymmetric model, we first consider the particle content in the model. In this model, three lepton superfield families are transformed as the triplet under the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ gauge group. We use the same notation for fermionic field content given in Refs. [25,26]

$$\hat{L}_l = \begin{pmatrix} \hat{\nu}_l \\ \hat{l} \\ \hat{l}^c \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{3}, 0), \quad l = e, \mu, \tau. \tag{2}$$

In parentheses it appears the transformation properties under the respective factors $(\mathrm{SU}(3)_C,\ \mathrm{SU}(3)_L,\ \mathrm{U}(1)_X)$.

In the quark sector, one quark superfield family is also put in the triplet representation of $\mathrm{SU}(3)_L$ as follows

$$\hat{Q}_{1L} = \begin{pmatrix} \hat{u}_1 \\ \hat{d}_1 \\ \hat{J} \end{pmatrix}_L \sim \left(\mathbf{3},\mathbf{3},\frac{2}{3}\right), \tag{3}$$

and their respective singlet quark superfields are given by

$$\hat{u}^c_{1L} \sim \left(\mathbf{3}^*,\mathbf{1},-\frac{2}{3}\right), \quad \hat{d}^c_{1L} \sim \left(\mathbf{3}^*,\mathbf{1},\frac{1}{3}\right), \quad \hat{J}^c_L \sim \left(\mathbf{3}^*,\mathbf{1},-\frac{5}{3}\right), \tag{4}$$

The remaining two quark generations are transformed as antitriplet superfield representation of $\mathrm{SU}(3)_L$ such as

$$\hat{Q}_{2L} = \begin{pmatrix} \hat{d}_2 \\ -\hat{u}_2 \\ \hat{j}_1 \end{pmatrix}_L , \quad \hat{Q}_{3L} = \begin{pmatrix} \hat{d}_3 \\ -\hat{u}_3 \\ \hat{j}_2 \end{pmatrix}_L \sim \left(\mathbf{3},\mathbf{3}^*,-\tfrac{1}{3}\right), \tag{5}$$

and their respective singlet superfields are transformed as follows

$$\begin{aligned} &\hat{u}^c_{2L}\ ,\hat{u}^c_{3L} \sim \left(\mathbf{3}^*,\mathbf{1},-\frac{2}{3}\right), \quad \hat{d}^c_{2L}\ ,\hat{d}^c_{3L} \sim \left(\mathbf{3}^*,\mathbf{1},\frac{1}{3}\right), \\ &\hat{j}^c_{1L}\ ,\hat{j}^c_{2L}\ \sim \left(\mathbf{3}^*,\mathbf{1},\frac{4}{3}\right)\ . \end{aligned} \tag{6}$$

The Eqs.(3,5) explain exactly the meaning of item 7 given at the introduction of this article.

On the other hand, the scalar superfields which are necessary to generate the fermion masses are

$$\hat{\rho} = \begin{pmatrix} \hat{\rho}^+ \\ \hat{\rho}^0 \\ \hat{\rho}^{++} \end{pmatrix} \sim (\mathbf{1},\mathbf{3},+1), \quad \hat{\chi} = \begin{pmatrix} \hat{\chi}^- \\ \hat{\chi}^{--} \\ \hat{\chi}^0 \end{pmatrix} \sim (\mathbf{1},\mathbf{3},-1). \tag{7}$$

To remove chiral anomalies generated by the superpartners of the scalars, we have to introduce two other scalar superfields as follows

$$\hat{\rho}' = \begin{pmatrix} \hat{\rho}'^{-} \\ \hat{\rho}'^{0} \\ \hat{\rho}'^{--} \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{3}^*, -1), \quad \hat{\chi}' = \begin{pmatrix} \hat{\chi}'^{+} \\ \hat{\chi}'^{++} \\ \hat{\chi}'^{0} \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{3}^*, +1). \tag{8}$$

It is to be noted that the superfields formalism is useful in writing the Lagrangian which is manifestly invariant under the supersymmetric transformations [37] with fermions and scalars put in chiral superfields while the gauge bosons in vector superfields. As usual, the superfield of a field ϕ will be denoted by $\hat{\phi}$ [38]. The chiral superfield of a multiplet ϕ is denoted by

$$\begin{aligned}\hat{\phi}(x,\theta,\bar{\theta}) = &\hat{\phi}(x) + i\,\theta\sigma^{\mu}\bar{\theta}\,\partial_{\mu}\hat{\phi}(x) + \frac{1}{4}\,\theta\theta\,\bar{\theta}\bar{\theta}\,\Box\hat{\phi}(x) \\ &+ \sqrt{2}\,\theta\phi(x) + \frac{i}{\sqrt{2}}\,\theta\theta\,\bar{\theta}\bar{\sigma}^{\mu}\partial_{\mu}\phi(x) \\ &+ \theta\theta\,F_{\phi}(x).\end{aligned} \tag{9}$$

Concerning the gauge bosons and their superpartners, if we denote the gluons by g^b the respective superparticles, the gluinos, are denoted by λ^b_C, with $b = 1, \ldots, 8$; and in the electroweak sector we have V^b, the gauge boson of $\mathrm{SU}(3)_L$, and their gaugino partners λ^b_A; finally we have the gauge boson of $U(1)_X$, denoted by $\hat{B}$, and its supersymmetric partner λ_B.

The vector superfield is given by

$$\begin{aligned}\hat{V}(x,\theta,\bar{\theta}) = &-\theta\sigma^{\mu}\bar{\theta}V_{\mu}(x) + i\theta\theta\overline{\theta\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) \\ &+ \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x).\end{aligned} \tag{10}$$

As the other version of the $\mathrm{SU}(3)_c \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$, the vector superfields for the gauge bosons of each factor $\mathrm{SU}(3)_C$, $\mathrm{SU}(3)_L$ and $\mathrm{U}(1)_X$ are denoted by $\hat{V}_C, \hat{\bar{V}}_C$; $\hat{V}, \hat{\bar{V}}$; and $\hat{V}'$, respectively, where we have defined

$$\begin{aligned}\hat{V}_C &= T^a \hat{V}^a_C, \quad \hat{\bar{V}}_C = \bar{T}^a \hat{V}^a{}_C, \quad a = 1, \cdots, 8; \\ \hat{V} &= T^a \hat{V}^a, \quad \hat{\bar{V}} = \bar{T}^a \hat{V}^a, \\ \hat{V}' &= T^9 \hat{B},\end{aligned} \tag{11}$$

where $T^a = \lambda^a/2$, $\bar{T}^a = -\lambda^{*a}/2$ are the generators of triplet and antitriplets representations, respectively, and λ^a are the Gell-Mann matrices, and the $T^9 = (1/\sqrt{6})\,\mathrm{diag}(1,\ 1,\ 1)$ is the generator of $\mathrm{U}(1)_X$ which satisfies the relation: $\mathrm{Tr}(T^aT^b) = 1/2\delta_{ab}$ with all $a, b = 1, 2, ..9$.

3 The Lagrangian

With the superfields introduced in the last section we can build an invariant supersymmetric Lagrangian. As usual as in supersymmetric model, for the model under consideration, we have

$$\mathcal{L}_{3-3-1} = \mathcal{L}_{\mathrm{SUSY}} + \mathcal{L}_{\mathrm{soft}}. \tag{12}$$

Here $\mathcal{L}_{\mathrm{SUSY}}$ is the supersymmetric piece, while $\mathcal{L}_{\mathrm{soft}}$ explicitly breaks supersymmetry. Below we will write each of these Lagrangians in terms of the respective superfields.

3.1 The supersymmetric terms

The supersymmetric terms can be divided as follows

$$\mathcal{L}_{\mathrm{SUSY}} = \mathcal{L}_{\mathrm{Lepton}} + \mathcal{L}_{\mathrm{Quarks}} + \mathcal{L}_{\mathrm{Gauge}} + \mathcal{L}_{\mathrm{Scalar}}, \tag{13}$$

where each term is given by

$$\mathcal{L}_{\mathrm{Lepton}} = \int d^4\theta\ \left[\hat{\bar{L}} e^{2g\hat{V}} \hat{L}\right], \tag{14}$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{Quarks}} = \int d^4\theta\ \Big[& \hat{\bar{Q}}_1 e^{2[g_s\hat{V}_c + g\hat{V} + (2g'/3)\hat{V}']}\hat{Q}_1 + \hat{\bar{Q}}_\alpha e^{2[g_s\hat{V}_c + g\hat{\bar{V}} - (g'/3)\hat{V}']}\hat{Q}_\alpha \\
& + \hat{\bar{u}}_i e^{2[g_s\hat{\bar{V}}_c - (2g'/3)\hat{V}']}\hat{u}_i + \hat{\bar{d}}_i e^{2[g_s\hat{\bar{V}}_c + (g'/3)\hat{V}']}\hat{d}_i \\
& + \hat{\bar{J}} e^{2[g_s\hat{\bar{V}}_c - (5g'/3)\hat{V}']}\hat{J} + \hat{\bar{j}}_i e^{2[g_s\hat{\bar{V}}_c + (4g'/3)\hat{V}']}\hat{j}_i \Big]
\end{aligned} \tag{15}$$

where the sum for $i = 1, 2, 3$, $\alpha = 1, 2$ and

$$\begin{aligned}
\mathcal{L}_{\mathrm{Gauge}} = \frac{1}{4} \times \Big[& \int d^2\theta\ \left(W_c^a W_c^a + W^a W^a + W' W'\right) \\
& + \int d^2\bar{\theta}\ \left(\bar{W}_c^a \bar{W}_c^a + \bar{W}^a \bar{W}^a + \bar{W}' \bar{W}'\right)\Big],
\end{aligned} \tag{16}$$

where $\hat{V}_c, \hat{\bar{V}}_c$, $\hat{V}$ and $\hat{\bar{V}}$ are defined in Eq.(11) and g_s, g and g' are the gauge couplings of $\mathrm{SU}(3)_C$, $\mathrm{SU}(3)_L$ and $\mathrm{U}(1)_X$, respectively. W_c^a, W^a and W' are the strength fields, and they are given by

$$
\begin{aligned}
W^a_{\alpha c} &= -\frac{1}{8g_s}\bar{D}\bar{D}e^{-2g_s\hat{V}_c}D_\alpha e^{-2g_s\hat{V}_c}, \\
W^a_{\alpha} &= -\frac{1}{8g}\bar{D}\bar{D}e^{-2g\hat{V}}D_\alpha e^{-2g\hat{V}}, \\
W'_{\alpha} &= -\frac{1}{4}\bar{D}\bar{D}D_\alpha\hat{V}' \ .
\end{aligned} \tag{17}
$$

Finally, the Lagrangian for the Higgs superfield is given as follows

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Scalar}} =& \int d^4\theta \ \left[\hat{\bar{\rho}}e^{2g\hat{V}+g'\hat{V}'}\hat{\rho} + \hat{\bar{\chi}}e^{2g\hat{V}-g'\hat{V}'}\hat{\chi} + \hat{\bar{\rho}}'e^{2g\hat{\bar{V}}-g'\hat{V}'}\hat{\rho}' + \hat{\bar{\chi}}'e^{2g\hat{\bar{V}}+g'\hat{V}'}\hat{\chi}'\right] \\
&+ \int d^2\theta\, W + \int d^2\bar{\theta}\,\bar{W},
\end{aligned} \tag{18}
$$

where W is the superpotential that is written details in the next subsection. After integrating the super-Lagrangian given in Eqs.(14,15,16) and Eq.(18), we obtain the Lagrangian given in Appendix A.

3.2 Superpotential.

Let us write the full superpotential in the model under consideration. The superpotential which is invariant under $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ group can be written by

$$
W = \frac{W_2}{2} + \frac{W_3}{3}, \tag{19}
$$

with W_2 is a combination of two chiral superfields and the terms permitted by the considered symmetry are

$$
W_2 = \mu_\rho\hat{\rho}\hat{\rho}' + \mu_\chi\hat{\chi}\hat{\chi}', \tag{20}
$$

and W_3 is invariant under the mentioned symmetry and a combination of three chiral superfields. That term has the following form

$$
W_3 = \lambda_1\epsilon\hat{L}\hat{L}\hat{L} + \lambda_2\epsilon\hat{L}\hat{\chi}\hat{\rho} + \sum_i \kappa_{1i}\hat{Q}_1\hat{\rho}'\hat{d}^c_i + \kappa_2\hat{Q}_1\hat{\chi}'\hat{J}^c
$$

$$
\begin{aligned}
&+\sum_{\alpha i}\kappa_{3\alpha i}\hat{Q}_\alpha\hat{\rho}\hat{u}^c_i+\sum_{\alpha\beta}\kappa_{4\alpha\beta}\hat{Q}_\alpha\hat{\chi}\hat{j}^c_\beta+\sum_{\alpha ij}\kappa_{5\alpha ij}\hat{Q}_\alpha\hat{L}_i\hat{d}^c_j \\
&+\sum_{i,j,k}\xi_{1ijk}\hat{d}^c_i\hat{d}^c_j\hat{u}^c_k+\sum_{ij\beta}\xi_{2ij\beta}\hat{u}^c_i\hat{u}^c_j\hat{j}^c_\beta+\sum_{i\beta}\xi_{3i\beta}\hat{d}^c_i\hat{J}^c\hat{j}^c_\beta,
\end{aligned}
\tag{21}
$$

with $i,j,k=1,2,3$, $\alpha=2,3$ and $\beta=1,2$. The terms κ_5 and ξ_2 will induce the proton decay as shown at [24].

Choosing, as we have done in [39], the following R-charges

$$
\begin{aligned}
&n_{\rho'}=-1,\;\; n_\rho=1,\;\; n_\chi=n_{\chi'}=0, \\
&n_L=n_{Q_i}=n_{d_i}=1/2,\;\; n_{J_i}=-1/2,\;\; n_u=-3/2,
\end{aligned}
\tag{22}
$$

it is easy to see that all the fields χ, χ', ρ, ρ', L, Q_i, u, d and J_i have R-charge equal to one, while their superpartners have opposite R-charge. This kind of symmetry is similar to that in the MSSM. The superpotential which satisfies the R- symmetry given in (22) can be written by

$$
\begin{aligned}
W=\;&\frac{\mu_\rho}{2}\hat{\rho}\hat{\rho}'+\frac{\mu_\chi}{2}\hat{\chi}\hat{\chi}'+\frac{1}{3}\Bigg[\lambda_1\epsilon\hat{L}\hat{L}\hat{L}+\lambda_2\epsilon\hat{L}\hat{\chi}\hat{\rho}+\sum_i\kappa_{1i}\hat{Q}_1\hat{\rho}'\hat{d}^c_i+\kappa_2\hat{Q}_1\hat{\chi}'\hat{J}^c \\
&+\sum_{\alpha i}\kappa_{3\alpha i}\hat{Q}_\alpha\hat{\rho}\hat{u}^c_i+\sum_{\alpha\beta}\kappa_{4\alpha\beta}\hat{Q}_\alpha\hat{\chi}\hat{j}^c_\beta+\sum_{\alpha ij}\kappa_{5\alpha ij}\hat{Q}_\alpha\hat{L}_i\hat{d}^c_j\Bigg]
\end{aligned}
\tag{23}
$$

Based on the superpotential given in Eq.(23), we can generate mass to neutrinos and recover all the nice consequences given at [39]. We will consider these details in the next section.

3.3 Breaked structure from SUSY RM 3-3-1 to $\mathrm{SU}(3)_C\otimes\mathrm{U}(1)_Q$.

The pattern of the symmetry breaking of the model is given by the following scheme (using the notation given at [39])

$$
\begin{aligned}
\text{SUSY RM 3-3-1}&\overset{\mathcal{L}_{\rm soft}}{\longmapsto}\mathrm{SU}(3)_C\otimes\mathrm{SU}(3)_L\otimes\mathrm{U}(1)_X \\
&\overset{\langle\chi\rangle\langle\chi'\rangle}{\longmapsto}\mathrm{SU}(3)_C\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y \\
&\overset{\langle\rho\rangle\langle\rho'\rangle}{\longmapsto}\mathrm{SU}(3)_C\otimes\mathrm{U}(1)_Q.
\end{aligned}
\tag{24}
$$

For the sake of simplicity, here we assume that vacuum expectation values (VEVs) are real. This means that the CP violation through the scalar exchange is not considered in this work. Note that non-supersymmetric 3-3-1 model with non real vev was studied at [7,8] and it is the point 5 given at introduction.

When one breaks the 3-3-1 symmetry to the $\mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Q$, the scalar fields get the following VEVs:

$$
\begin{aligned}
<\rho>&=\begin{pmatrix} 0 \\ u \\ 0 \end{pmatrix}, \quad <\chi>=\begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix}, \\
<\rho'>&=\begin{pmatrix} 0 \\ u' \\ 0 \end{pmatrix}, \quad <\chi'>=\begin{pmatrix} 0 \\ 0 \\ w' \end{pmatrix},
\end{aligned}
\tag{25}
$$

where $u = v_\rho/\sqrt{2}$, $w = v_\chi/\sqrt{2}$, $u' = v_{\rho'}/\sqrt{2}$ and $w' = v_{\chi'}/\sqrt{2}$. Because of the pattern of the symmetry breaking given in (24), the VEV's of the model under consideration have to be satisfied the conditions:

$$
w, w' \gg u, u'. \tag{26}
$$

On the other hand, the constraint on the W bosons mass [26], see Eq.(37), we get the following constraint on V_ρ^2

$$
V_\rho^2 = (246 \text{ GeV})^2 \tag{27}
$$

where $V_\rho^2 = v_\rho^2 + v_\rho'^2$.

3.4 Soft terms

The most general soft supersymmetry breaking terms, which do not induce quadratic divergence, are described by Girardello and Grisaru [40]. They found that the allowed terms can be categorized as follows:

- The scalar mass term

$$
\mathcal{L}_{\mathrm{SMT}} = -m^2 A^\dagger A, \tag{28}
$$

- The gaugino mass term

$$
\mathcal{L}_{\mathrm{GMT}} = -\frac{1}{2}(M_\lambda \lambda^a \lambda^a + \mathrm{H.c.}) \tag{29}
$$

- The scalar interaction terms

$$\mathcal{L}_{\rm int} = m_{ij}A_iA_j + f_{ijk}\epsilon^{ijk}A_iA_jA_k + {\rm H.c.} \tag{30}$$

The soft SUSY breaking parameters are in general complex and they also can generate SUSY flavor problem. Therefore we can expect that in this model, there are several sources of CP violation as well as flavor problem. This subject can be explored in the future.

In the model, the soft terms must be consistent with the 3-3-1 gauge symmetry. Hence, the soft terms have the following form

$$\mathcal{L}_{\rm soft} = \mathcal{L}_{\rm GMT} + \mathcal{L}_{\rm SMT} + \mathcal{L}_{\rm int}, \tag{31}$$

where

$$\mathcal{L}_{\rm GMT} = -\frac{1}{2}\left[m_{\lambda_C}\sum_{a=1}^{8}(\lambda^a_C\lambda^a_C) + m_\lambda\sum_{a=1}^{8}(\lambda^a_A\lambda^a_A) + m'\lambda_B\lambda_B + {\rm H.c.}\right], \tag{32}$$

where λ_C are the gluinos, λ_A are the gauginos of SU(3) and λ_B is the gauginos of U(1) [see Eq.(A.16)]. The gauginos get their masses at SUSY broken scale while their superpartners (the gauge bosons) are massless at this scale, because their masses appear only after we break the symmetry $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ [see Eq.(37)] in next section. The second term which gains masses to all the scalars is written as

$$\begin{aligned}\mathcal{L}_{\rm SMT} = & -m^2_\rho\rho^\dagger\rho - m^2_\chi\chi^\dagger\chi - m^2_{\rho'}\rho'^\dagger\rho' - m^2_{\chi'}\chi'^\dagger\chi' \\ & - m^2_L\hat{L}^\dagger_{aL}\hat{L}_{aL} - m^2_{Q_\alpha}\hat{Q}^\dagger_{\alpha L}\hat{Q}_{\alpha L} - m^2_{Q_3}\hat{Q}^\dagger_{3L}\hat{Q}_{3L} \\ & - m^2_{u_i}\hat{u}^{c\dagger}_{iL}\hat{u}^c_{iL} - m^2_{d_i}\hat{d}^{c\dagger}_{iL}\hat{d}^c_{iL} - m^2_J\hat{J}^{c\dagger}_L\hat{J}^c_L - m^2_{j_\beta}\hat{j}^{c\dagger}_{\beta L}\hat{j}^c_{\beta L}\end{aligned} \tag{33}$$

and the last term is given by

$$\begin{aligned}\mathcal{L}_{\rm int} = & \Big[\varepsilon_{0abc}\epsilon\hat{L}_{aL}\hat{L}_{bL}\hat{L}_{cL} + \varepsilon_{1ab}\epsilon\hat{L}_{aL}\chi\rho + \hat{Q}_{\alpha L}\Big(\omega_{1\alpha i}\rho\hat{u}^c_{iL} + \omega_{3\alpha aj}\hat{L}_{aL}\hat{d}^c_{jL} \\ & + \omega_{4\alpha\beta}\chi\hat{j}^c_{\beta L}\Big) + \hat{Q}_{3L}(\zeta_{1i}\rho'\hat{d}^c_{iL} + \zeta_{3J}\chi'\hat{J}^c_L) + \varsigma_{1ijk}\hat{d}^c_{iL}\hat{d}^c_{jL}\hat{u}^c_{kL} \\ & + \varsigma_{2i\beta}\hat{d}^c_{iL}\hat{J}^c_L\hat{j}^c_{\beta L} + \varsigma_{3ij\beta}\hat{u}^c_{iL}\hat{u}^c_{jL}\hat{j}^c_{\beta L} + {\rm H.c.}\Big].\end{aligned} \tag{34}$$

4 Gauge boson masses

Just as it did in the usual 3-3-1 model [2,26,39], we can divide the gauge boson masses into two parts namely the charged and neutral gauge boson masses. The mass Lagrangian for the gauge bosons can be obtained by

$$
\begin{aligned}
\mathcal{L}^{\text{gauge}}_{\text{mass}} = &\left(0\ 0\ \tfrac{v_\chi}{\sqrt{2}} \right) \left(\frac{g}{2}\lambda^a V^\mu_a - \frac{g'}{\sqrt{6}} B^\mu \right)^2 \left(0\ 0\ \tfrac{v_\chi}{\sqrt{2}} \right)^T \\
&+ \left(0\ 0\ \tfrac{v_{\chi'}}{\sqrt{2}} \right) \left(-\frac{g}{2}\lambda^{*a} V^\mu_a + \frac{g'}{\sqrt{6}} B^\mu \right)^2 \left(0\ 0\ \tfrac{v_{\chi'}}{\sqrt{2}} \right)^T \\
&+ \left(0\ \tfrac{v_\rho}{\sqrt{2}}\ 0 \right) \left(\frac{g}{2}\lambda^a V^\mu_a + \frac{g'}{\sqrt{6}} B^\mu \right)^2 \left(0\ \tfrac{v_\rho}{\sqrt{2}}\ 0 \right)^T \\
&+ \left(0\ \tfrac{v_{\rho'}}{\sqrt{2}}\ 0 \right) \left(-\frac{g}{2}\lambda^{*a} V^\mu_a - \frac{g'}{\sqrt{6}} B^\mu \right)^2 \left(0\ \tfrac{v_{\rho'}}{\sqrt{2}}\ 0 \right)^T . \qquad (35)
\end{aligned}
$$

The Lagrangian in Eq.(35) produces the charged gauge boson mass terms given as follows

$$
\mathcal{L}^{\text{charged}}_{\text{mass}} = M^2_W W^-_\mu W^{+\mu} + M^2_V V^-_\mu V^{+\mu} + M^2_U U^{--}_\mu U^{++\mu}, \qquad (36)
$$

with

$$
\begin{aligned}
M^2_U &= \frac{g^2}{8}(v^2_\rho + v^2_\chi + v^2_{\rho'} + v^2_{\chi'}), \\
M^2_W &= \frac{g^2}{8}(v^2_\rho + v^2_{\rho'}), \\
M^2_V &= \frac{g^2}{8}(v^2_\chi + v^2_{\chi'}) \qquad (37)
\end{aligned}
$$

and the mass eigenvectors are given respectively

$$
\begin{aligned}
W^\pm_\mu(x) &= \frac{1}{\sqrt{2}}\left[V^1_\mu(x) \mp i V^2_\mu(x) \right], \\
V^\pm_\mu(x) &= \frac{1}{\sqrt{2}}\left[V^4_\mu(x) \pm i V^5_\mu(x) \right], \\
U^{\pm\pm}_\mu(x) &= \frac{1}{\sqrt{2}}\left[V^6_\mu(x) \pm i V^7_\mu(x) \right]. \qquad (38)
\end{aligned}
$$

The neutral gauge bosons $(V^\mu_3, V^\mu_8, B^\mu)$ are mixing. The mass Lagrangian for neutral gauge bosons are given as

$$
\mathcal{L}^{\text{neutral}}_{\text{mass}} = \left(V^\mu_3\ V^\mu_8\ B^\mu \right) M^2_{NG} \left(V_{3\mu}\ V_{8\mu}\ B_\mu \right)^T \qquad (39)
$$

with

$$M_{\mathrm{NG}}^2 = \frac{g^2}{4} \begin{pmatrix} \frac{v_\rho^2+v_{\rho'}^2}{2} & -\frac{v_\rho^2+v_{\rho'}^2}{\sqrt{3}} & -t\sqrt{\frac{2}{3}}(v_\rho^2+v_{\rho'}^2) \\ -\frac{v_\rho^2+v_{\rho'}^2}{\sqrt{3}} & \frac{1}{3}\left(v_\rho^2+v_{\rho'}^2+4v_\chi^2+4v_{\chi'}^2\right) & \frac{2t}{3\sqrt{2}}(v_\rho^2+v_{\rho'}^2+2v_\chi^2+2v_{\chi'}^2) \\ -t\sqrt{\frac{2}{3}}(v_\rho^2+v_{\rho'}^2) & \frac{2t}{3\sqrt{2}}(v_\rho^2+v_{\rho'}^2+2v_\chi^2+2v_{\chi'}^2) & \frac{2t^2}{3}(v_\rho^2+v_{\rho'}^2+v_\chi^2+v_{\chi'}^2) \end{pmatrix}. \tag{40}$$

After diagonalization the matrix M_{NG}^2, we obtain the mass eigenvalues as follows

$$\begin{aligned} M_\gamma^2 &= 0, \\ M_Z^2 &= \frac{g^2(2+t^2)}{24}\left(v_\rho^2+v_{\rho'}^2+v_\chi^2+v_{\chi'}^2\right. \\ &\quad \left. -\sqrt{\frac{-4(3+2t^2)}{2+t^2}(v_\rho^2+v_{\rho'}^2)(v_\chi^2+v_{\chi'}^2)+(v_\rho^2+v_{\rho'}^2+v_\chi^2+v_{\chi'}^2)^2}\right), \\ M_{Z'}^2 &= \frac{g^2(2+t^2)}{24}\left(v_\rho^2+v_{\rho'}^2+v_\chi^2+v_{\chi'}^2\right. \\ &\quad \left. +\sqrt{\frac{-4(3+2t^2)}{2+t^2}(v_\rho^2+v_{\rho'}^2)(v_\chi^2+v_{\chi'}^2)+(v_\rho^2+v_{\rho'}^2+v_\chi^2+v_{\chi'}^2)^2}\right) \end{aligned} \tag{41}$$

and the mass eigenvectors, respectively:

$$\begin{aligned} A^\mu &= \frac{1}{\sqrt{1+\frac{2t^2}{3}}}\left(\frac{t}{\sqrt{6}}V_3^\mu - \frac{t}{\sqrt{2}}V_8^\mu + B^\mu\right), \\ Z^\mu &= -\frac{\sqrt{3}(c_\varsigma + \frac{s_\varsigma}{\sqrt{3+2t^2}})}{2}V_3^\mu - \frac{-c_\varsigma + \frac{3s_\varsigma}{\sqrt{3+2t^2}}}{2}V_8^\mu + \frac{\sqrt{2}ts_\varsigma}{\sqrt{3+2t^2}}B^\mu, \\ Z'^\mu &= -\frac{\sqrt{3}(-s_\varsigma + \frac{c_\varsigma}{\sqrt{3+2t^2}})}{2}V_3^\mu - \frac{s_\varsigma + \frac{3c_\varsigma}{\sqrt{3+2t^2}}}{2}V_8^\mu - \frac{\sqrt{2}tc_\varsigma}{\sqrt{3+2t^2}}B^\mu, \end{aligned}$$

with t and ς are defined as follows

$$t = \frac{g'}{g} \equiv \frac{6\sin^2\theta_W}{1-4\sin^2\theta_W}, \tag{42}$$

$$\tan(2\varsigma) = \frac{\sqrt{3+2t^2}}{1+t^2}\left(\frac{v_\chi^2+v_{\chi'}^2-v_\rho^2-v_{\rho'}^2}{v_\chi^2+v_{\chi'}^2+v_\rho^2+v_{\rho'}^2}\right). \tag{43}$$

The relation in (42) predicts that there exists an energy scale at which the model loses its pertubative character as we have noted at the main aspect of the 3-3-1 models. Therefore, in order to keep its pertubative character, we have $\sin^2\theta_W(\mu) < 1/4$ at any energy scale.

Let us summary the gauge mass spectrum. The gauge boson mixing is separated into two parts. One is charged gauge bosons and one is neutral gauge bosons. The exact eigensvectors and eigenvalues are obtained. According to the limit given in (26), we get the constraint on the gauge mass as follows

$$M_{Z'} > M_U > M_V > M_Z > M_W. \tag{44}$$

This constraint is similar to those in [39]. As the masses of all new gauge masses are proportional to v_χ and $v_{\chi'}$, both are in the TeV scale [see Eq.(24)]. It explains why the new gauge bosons have not been yet detected, but their masses can be discovered by the experiments at the Large Hadron Collider (LHC) and at the International Linear Collider (ILC).

5 Fermion mass matrices

In this section we will show that all the fermions of this model get masses in concordance with the experimental data.

5.1 Doubly charged charginos

As in previous works [26,39], we get the same result without any modification. These new states can be dicovered in the LHC throughout the following

$$\bar{p} + p \longrightarrow \tilde{\chi}^{++}\tilde{\chi}^{--}. \tag{45}$$

The similar one [39]

$$e^- + e^- \longrightarrow \tilde{\chi}^{--}\tilde{\chi}^{0}, \tag{46}$$

is a prospective reaction in the ILC.

5.2 Charged leptons and charginos

Let us consider the mass spectrum of the charged leptons and charginos. In the model under consideration, mass mixing matrix of the charged leptons and charginos is similar to that given in [29,30,36]. In our case, the e, μ and τ leptons gain mass without a sextet Higgs or the charged lepton singlet. Note

that the Higgsinos $\tilde{\rho}, \tilde{\chi}$ and their respective primed fields have the same charge assignment of the triplets ρ and χ. Hence, they can mix with the usual leptons.

Let us first consider the charged lepton and chargino masses. Denoting

$$\begin{aligned} \phi^{+} &= (e^{c}, \mu^{c}, \tau^{c}, -i\lambda^{+}_{W}, -i\lambda^{+}_{V}, \tilde{\rho}^{+}, \tilde{\chi}'^{+}, \tilde{\rho'}^{+}, \tilde{\chi}^{+})^{T}, \\ \phi^{-} &= (e, \mu, \tau, -i\lambda^{-}_{W}, -i\lambda^{-}_{V}, \tilde{\rho}'^{-}, \tilde{\chi}^{-}, \tilde{\rho'}^{-}, \tilde{\chi}^{-})^{T}, \end{aligned} \quad (47)$$

where all the fermionic fields are still Weyl spinors, we can also, as before, define $\Psi^{\pm} = (\phi^{+}\phi^{-})^{T}$. Then, the mass term is written in the form $-(1/2)[\Psi^{\pm T} Y^{\pm} \Psi^{\pm} +$ H.c.] where $Y^{\pm}$ is given by:

$$Y^{\pm} = \begin{pmatrix} 0 & X^{T} \\ X & 0 \end{pmatrix}, \quad (48)$$

with X matrix is defined as

$$X = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\lambda_{2e}}{3}w & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\lambda_{2\mu}}{3}w & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\lambda_{2\tau}}{3}w & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{\lambda} & 0 & gu & 0 & -gu' & 0 \\ 0 & 0 & 0 & 0 & m_{\lambda} & 0 & -gw' & 0 & gw \\ \frac{\lambda_{2e}}{3}w & \frac{\lambda_{2\mu}}{3}w & \frac{\lambda_{2\tau}}{3}w & gu & 0 & 0 & 0 & -\frac{\mu_{\rho}}{2} & 0 \\ 0 & 0 & 0 & 0 & -gw' & 0 & 0 & 0 & -\frac{\mu_{\chi}}{2} \\ 0 & 0 & 0 & -gu' & 0 & -\frac{\mu_{\rho}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & gw & 0 & -\frac{\mu_{\chi}}{2} & 0 & 0 \end{pmatrix}, \quad (49)$$

This mass matrix gives two zero eigenvalues [36]. One of two zero eigenvalues is identified to the electron mass and the remaining one is identified to the muon mass. It means that the electron and muon are massless at the tree level. If there is not a discrete symmetry, which is added to the Lagrangian, the charged lepton can get a mass by loop corrections as done at [41].

In this model, the electron still couples with the gaugino λ_{B} of U(1) group, [see Eq.(A.4)], in a similar way as shown at [41]. As the selectrons and the gauginos get their masses due the soft terms given in Eqs.(32,33), it allows us to draw the diagram of Fig.1 that gives contribution to the electron mass. Therefore, at the one loop correction the electron mass is given by:

$$
\begin{aligned}
m_e \propto & \frac{\alpha_{\mathrm{U}(1)} \sin(2\theta_{\tilde{e}})}{\pi} m' \left[\frac{m^2_{\tilde{e}_1}}{m^2_{\tilde{e}_1} - m'^2} \ln \left(\frac{m^2_{\tilde{e}_1}}{m'^2} \right) \right. \\
& \left. - \frac{m^2_{\tilde{e}_2}}{m^2_{\tilde{e}_2} - m'^2} \ln \left(\frac{m^2_{\tilde{e}_2}}{m'^2} \right) \right] \quad , \\
m_\mu \propto & \frac{\alpha_{\mathrm{U}(1)} \sin(2\theta_{\tilde{\mu}})}{\pi} m' \left[\frac{m^2_{\tilde{\mu}_1}}{m^2_{\tilde{\mu}_1} - m'^2} \ln \left(\frac{m^2_{\tilde{\mu}_1}}{m'^2} \right) \right. \\
& \left. - \frac{m^2_{\tilde{\mu}_2}}{m^2_{\tilde{\mu}_2} - m'^2} \ln \left(\frac{m^2_{\tilde{\mu}_2}}{m'^2} \right) \right] \quad ,
\end{aligned}
\tag{50}
$$

where $m', m_{\tilde{e}}, m_{\tilde{\mu}}$ are soft parameters given in Eq. (32), $\alpha_{\mathrm{U}(1)} = g'^2/(4\pi)$ and the $\theta_{\tilde{e}}, \theta_{\tilde{\mu}}$ are defined in Eq.(63). As we expect the smuon is heavier than the selectron, it explain why the muon is heavier than electron, see at SPS scenarios [42,43,44].

The tau gets its mass at tree level, it explains why the tau is heavier than the muon and electron. The other four mass values are at GeV scale as shown at [36]. The way we perform the diagonalization, as well, the particle definitions are given at [29,30,36].

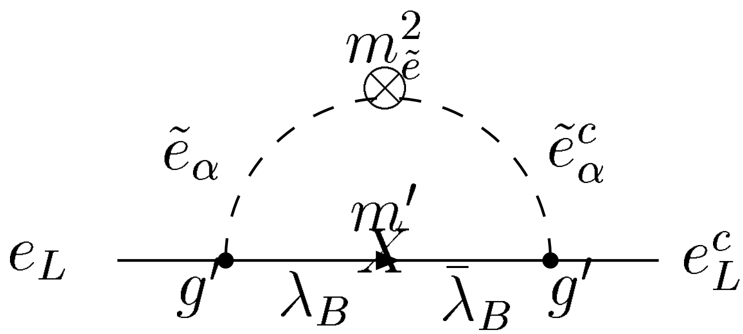

Fig. 1. Diagram giving mass to electron which does not appear in the superpotential, $\tilde{e}$ is the selectron and the label $\alpha = 1, 2$. The diagram to the muon is similar (just change the selecton by the smuon)

5.3 Neutrinos and neutralinos

Because the existence of the interaction between neutrinos and neutralinos, their mass matrix has a mixture. The mass term in the basis

$$
\Psi^0 = \left(\nu_e \ \nu_\mu \ \nu_\tau \ -i\lambda^3_A \ -i\lambda^8_A \ -i\lambda_B \ \tilde{\rho}^0 \ \tilde{\rho}'^0 \ \tilde{\chi}^0 \ \tilde{\chi}'^0 \right)^T \tag{51}
$$

is given by $-(1/2)[(\Psi^0)^T Y^0 \Psi^0 + \text{H.c.}]$, where

$$
Y^0 = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_{2e}}{3}w & 0 & \frac{\lambda_{2e}}{3}u & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_{2\mu}}{3}w & 0 & \frac{\lambda_{2\mu}}{3}u & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_{2\tau}}{3}w & 0 & \frac{\lambda_{2\tau}}{3}u & 0 \\
0 & 0 & 0 & m_\lambda & 0 & 0 & -\frac{gu}{\sqrt{2}} & \frac{gu'}{\sqrt{6}} & 0 & 0 \\
0 & 0 & 0 & 0 & m_\lambda & 0 & \frac{gu}{\sqrt{6}} & -\frac{gu'}{\sqrt{6}} & \frac{gw}{\sqrt{6}} & -\frac{gw'}{\sqrt{6}} \\
0 & 0 & 0 & 0 & 0 & m' & \frac{g'u}{\sqrt{6}} & -\frac{g'u'}{\sqrt{6}} & -\frac{g'w}{\sqrt{6}} & \frac{g'w'}{\sqrt{6}} \\
\frac{\lambda_{2e}}{3}w & \frac{\lambda_{2\mu}}{3}w & \frac{\lambda_{2\tau}}{3}w & -\frac{gu}{\sqrt{2}} & \frac{gu}{\sqrt{6}} & \frac{g'u}{\sqrt{2}} & 0 & -\frac{\mu_\rho}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{gu'}{\sqrt{2}} & -\frac{gu'}{\sqrt{6}} & -\frac{g'u'}{\sqrt{6}} & -\frac{\mu_\rho}{2} & 0 & 0 & 0 \\
\frac{\lambda_{2e}}{3}u & \frac{\lambda_{2\mu}}{3}u & \frac{\lambda_{2\tau}}{3}u & 0 & \frac{gw}{\sqrt{6}} & \frac{g'w}{\sqrt{6}} & 0 & 0 & 0 & -\frac{\mu_\chi}{2} \\
0 & 0 & 0 & 0 & -\frac{gw'}{\sqrt{6}} & \frac{g'w'}{\sqrt{6}} & 0 & 0 & -\frac{\mu_\chi}{2} & 0
\end{pmatrix}. \tag{52}
$$

This matrix has two zero eigenvalues. The parameter m_λ is defined in Eq. (32). On the other hand, the electron's neutrinos still couples with the selectron and down squark, while the muon's neutrinos couple with smuons and strange squark. These couplings lead to the diagrams shown in Fig.2. These diagrams

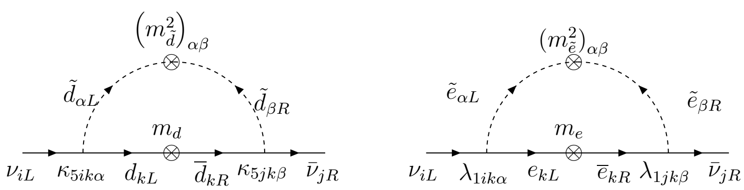

Fig. 2. Diagrams giving masses to electron's and muon's neutrinos which do not appear in the superpotential, $\tilde{e}$ is the selectron and $\tilde{d}$ is the down squark and the label $\alpha = 1, 2$.

give the contribution to the neutrino mass given in Eq.(53).

$$
\begin{aligned}
m_{\nu_e} \simeq \frac{1}{8\pi^2} \sum_{\imath=1}^{2} \Bigg\{ & m' \left[\lambda_{1e11}\lambda_{1e11} \frac{m^2_{\tilde{e}_1}}{m^2_{\tilde{e}_1} - m'^2} \ln\left(\frac{m^2_{\tilde{e}_1}}{m'^2} \right) \right. \\
& \left. - \lambda_{1e12}\lambda_{1e12} \frac{m^2_{\tilde{e}_2}}{m^2_{\tilde{e}_2} - m'^2} \ln\left(\frac{m^2_{\tilde{e}_2}}{m'^2} \right) \right] \\
& + 3m_{\tilde{g}} \left[\kappa_{5e11}\kappa_{5e11} \frac{m^2_{\tilde{d}_1}}{m^2_{\tilde{d}_1} - m'^2} \ln\left(\frac{m^2_{\tilde{d}_1}}{m^2_{\tilde{g}}} \right) \right. \\
& \left. - \kappa_{5e12}\kappa_{5e12} \frac{m^2_{\tilde{d}_2}}{m^2_{\tilde{d}_2} - m'^2} \ln\left(\frac{m^2_{\tilde{d}_2}}{m^2_{\tilde{g}}} \right) \right] \Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
m_{\nu_\mu} \simeq \frac{1}{8\pi^2}\sum_{\imath=1}^{2}\Bigg\{ & m'\left[\lambda_{1\mu21}\lambda_{1\mu21}\frac{m^2_{\tilde{\mu}_1}}{m^2_{\tilde{\mu}_1}-m'^2}\ln\left(\frac{m^2_{\tilde{\mu}_1}}{m'^2}\right)\right.\\
& \left. -\lambda_{1\mu22}\lambda_{1\mu22}\frac{m^2_{\tilde{\mu}_2}}{m^2_{\tilde{\mu}_2}-m'^2}\ln\left(\frac{m^2_{\tilde{\mu}_2}}{m'^2}\right)\right]\\
& +3m_{\tilde{g}}\left[\kappa_{5\mu21}\kappa_{5\mu21}\frac{m^2_{\tilde{s}_1}}{m^2_{\tilde{s}_1}-m'^2}\ln\left(\frac{m^2_{\tilde{s}_1}}{m^2_{\tilde{g}}}\right)\right.\\
& \left. -\kappa_{5\mu22}\kappa_{5\mu22}\frac{m^2_{\tilde{s}_2}}{m^2_{\tilde{s}_2}-m'^2}\ln\left(\frac{m^2_{\tilde{s}_2}}{m^2_{\tilde{g}}}\right)\right]\Bigg\},
\end{aligned}
\tag{53}
$$

where $m_{\tilde{g}}$ is the mass of a gluino, $m_{\tilde{d}}$ is the down-squark mass and $m_{\tilde{s}}$ is the strange-squark mass.

The neutrinos masses are proportional to λ_1, κ_5, the parameters which break the lepton number conservation. The electron mass is proportional to g. Due to this fact we expect that it must be satisfied a condition $\lambda_1, \kappa_5 \ll g'$, then we can explain the reason why neutrinos are much lighter than the charged leptons.

5.4 Quarks

Let us first consider the u-quark type. First, we define the basis as done in Ref. [39], particularly

$$
\psi^+_u = \begin{pmatrix} u_1 \ u_2 \ u_3 \end{pmatrix}^T, \quad \psi^-_u = \begin{pmatrix} u^c_1 \ u^c_2 \ u^c_3 \end{pmatrix}^T, \tag{54}
$$

where all the u-quark fields are still Weyl spinors, we can also define $\Psi^\pm_u = (\psi^+_u \psi^-_u)^T$. Then, the mass term is written in the form

$$
-(1/2)[\Psi^{\pm T}_u Y^\pm_u \Psi^\pm_u + \text{H.c.}].
$$

Here $Y^\pm_u$ is given by

$$
Y^\pm_u = \begin{pmatrix} 0 & X^T_u \\ X_u & 0 \end{pmatrix}, \tag{55}
$$

with

$$
X_u = \frac{1}{3}\begin{pmatrix} \kappa_{311}u & -\kappa_{312}u & 0 \\ -\kappa_{321}u & \kappa_{322}u & 0 \\ -\kappa_{331}u & \kappa_{323}u & 0 \end{pmatrix}, \tag{56}
$$

where the VEVs are defined in Eq.(25). The mass spectrum of the up quarks contains one massless particle. However the lightest quark will get mass due its coupling to gluino as shown in Fig.3. Therefore the up quark's mass is given by

$$m_u \propto \frac{\alpha_s \sin(2\theta_{\tilde{u}})}{\pi} m_{\tilde{g}} \left[\frac{M^2_{\tilde{u}_1}}{M^2_{\tilde{u}_1} - m^2_{\tilde{g}}} \ln\left(\frac{M^2_{\tilde{u}_1}}{m^2_{\tilde{g}}}\right) - \frac{M^2_{\tilde{u}_2}}{M^2_{\tilde{u}_2} - m^2_{\tilde{g}}} \ln\left(\frac{M^2_{\tilde{u}_2}}{m^2_{\tilde{g}}}\right)\right] \quad (57)$$

where $\alpha_s = g_s^2/(4\pi)$, $m_{\tilde{g}}$ and $m_{\tilde{u}}$ are the masses of the gluino and up-squark, respectively, and $\theta_{\tilde{u}}$ is the mixing angle of left- and right-handed up-squarks given in Eq.(63).

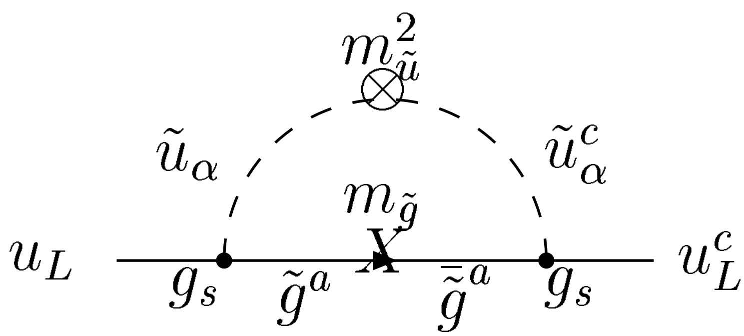

Fig. 3. Diagram giving mass to up quark which does not appear in the superpotential, $\tilde{u}$ is the up-squark.

Let us consider the d-quark type. Doing similarly as in the up-quark sector, we define $\Psi^{\pm}_d = (\psi^+_d \psi^-_d)^T$, then the mass term is written in the form $-(1/2)[\Psi^{\pm T}_d Y^{\pm}_d \Psi^{\pm}_d + \text{H.c.}]$ where $Y^{\pm}_d$ is given by

$$Y^{\pm}_d = \begin{pmatrix} 0 & X^T_d \\ X_d & 0 \end{pmatrix}, \quad (58)$$

with

$$X_d = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \kappa_{11}u' & \kappa_{12}u' & \kappa_{13}u' \end{pmatrix}, \quad (59)$$

In this sector, there are two massless eigenvalues. We can implement the same mechanism analyzed at [45] to give mass to d and s quarks. Thus, the model under consideration is compatible with chiral theory.

Analogously, looking at Fig.4 and Eq.(63), we get the expression for mass of d-quark [41]

$$m_d \propto \frac{\alpha_s \sin(2\theta_{\tilde{d}})}{\pi} m_{\tilde{g}} \left[\frac{M^2_{\tilde{d}_1}}{M^2_{\tilde{d}_1} - m^2_{\tilde{g}}} \ln \left(\frac{M^2_{\tilde{d}_1}}{m^2_{\tilde{g}}} \right) - \frac{M^2_{\tilde{d}_2}}{M^2_{\tilde{d}_2} - m^2_{\tilde{g}}} \ln \left(\frac{M^2_{\tilde{d}_2}}{m^2_{\tilde{g}}} \right) \right] . \quad (60)$$

For the s-quark, looking at Fig.5 and Eq.(65), we obtain [45]

$$\begin{aligned} m_s = \frac{\alpha_s m_{\tilde{g}}}{4\pi^3} \sum_{\alpha=1}^{2} \Bigg\{ & R^{(d)}_{1\alpha} R^{(d)}_{2\alpha} \frac{m^2_{\tilde{g}}}{(m^2_{\tilde{g}} - m^2_{\tilde{d}_\alpha})} \ln \left(\frac{m^2_{\tilde{g}}}{m^2_{\tilde{d}_\alpha}} \right) \\ & + R^{(d)}_{1\alpha+2} R^{(d)}_{2\alpha+2} \frac{m^2_{\tilde{g}}}{(m^2_{\tilde{g}} - m^2_{\tilde{d}_{\alpha+2}})} \ln \left(\frac{m^2_{\tilde{g}}}{m^2_{\tilde{d}_{\alpha+2}}} \right) \\ & + \frac{R^{(d)}_{1\alpha} R^{(d)}_{2\alpha+2}}{(m^2_{\tilde{d}_\alpha} - m^2_{\tilde{d}_{\alpha+2}})(m^2_{\tilde{g}} - m^2_{\tilde{d}_\alpha})(m^2_{\tilde{d}_{\alpha+2}} - m^2_{\tilde{g}})} \\ & \times \left(\delta^d_{12} \right)_{LR} M^2_{SUSY} \Bigg[m^2_{\tilde{d}_\alpha} m^2_{\tilde{d}_{\alpha+2}} \ln \left(\frac{m^2_{\tilde{d}_\alpha}}{m^2_{\tilde{d}_{\alpha+2}}} \right) \\ & + m^2_{\tilde{d}_\alpha} m^2_{\tilde{g}} \ln \left(\frac{m^2_{\tilde{g}}}{m^2_{\tilde{d}_\alpha}} \right) + m^2_{\tilde{d}_{\alpha+2}} m^2_{\tilde{g}} \ln \left(\frac{m^2_{\tilde{d}_{\alpha+2}}}{m^2_{\tilde{g}}} \right) \Bigg] \Bigg\} , \end{aligned} \quad (61)$$

where $\theta_{\tilde{d}}$ are mixing angles, $R^d_{\beta\alpha}$ is defined at Eq.(65) and $m^2_{\tilde{d}_\alpha}$ are the eigenvalues of Eq.(64) and they are the physical masses of $\tilde{s}_1, \tilde{s}_2$, $\tilde{b}_1$ and $\tilde{b}_2$.

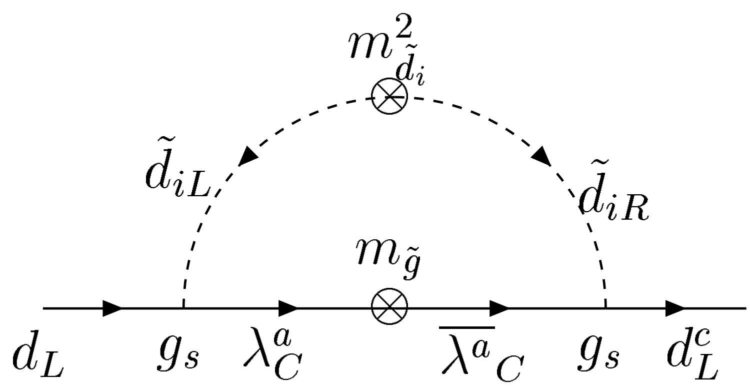

Fig. 4. Diagram giving mass to quark d which does not appear in the superpotential, $\tilde{g}$ is the gluino, $\tilde{d}_i$, $i = 1, 2$, is the down squark.

The electron mass is given by Eq.(50), the mass of up quarks is given by Eq.(57) and the down quark is given by Eq.(60). Note that the quark masses are proportional to g_s while lepton masses are proportional to g. The fact that

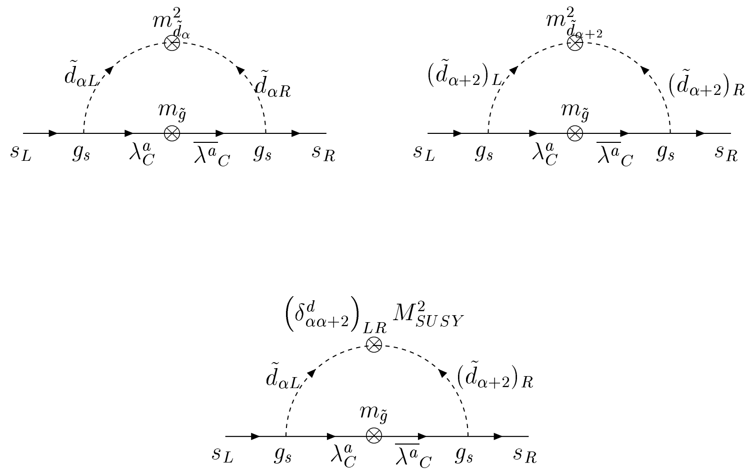

Fig. 5. Diagram giving mass to s quark which does not appear in the superpotential, $\tilde{g}$ is the gluino, $\tilde{s}_i$ and $\tilde{b}_i$, $i = 1, 2$, are the squark s and sbottom, respectively.

$g_s \gg g$ gives an explanation why quarks are heavier than the leptons. The mass of s-quark is given by Eq.(61), comparing this formula with Eq.(60) we can explain why s-quark is heavier than d-quark.

5.5 Sfermions

It is known that in the general case, the sfermions have a flavor mixing. It leads to all sfermions mass matrices are 6×6 matrices [45]. Therefore the slepton sector contains lepton flavor violation at the tree level. In order to avoid this problem we will neglect the generation mixing in the slepton sector. This assumption is not held for all other squark sectors. Each 6×6 slepton mass matrix can be divided into three 2×2 mass matrices. The off diagonal left-right mixing is proportional to the fermion masses.

Here, we will only present the main formulas. In the case of charged sleptons we can generally write 2×2 mass matrices

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} m^2_{\tilde{f}_L} & a_f m_f \\ a_f m_f & m^2_{\tilde{f}_R} \end{pmatrix} = (\mathcal{R}^{\tilde{f}}) \begin{pmatrix} m^2_{\tilde{f}_1} & 0 \\ 0 & m^2_{\tilde{f}_2} \end{pmatrix} \mathcal{R}^{\tilde{f}}, \qquad (62)$$

where $\tilde{f} = \tilde{e},\ \tilde{\mu},\ \tilde{\tau}$. The weak eigenstates $\tilde{f}_L$ and $\tilde{f}_R$ are thus related to their mass eigenstates $\tilde{f}_1$ and $\tilde{f}_2$, where $\tilde{f}_1$ is the lighter sfermion, by

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} = \mathcal{R}^{\tilde{f}} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}, \qquad \mathcal{R}^{\tilde{f}} = \begin{pmatrix} \cos\theta_{\tilde{f}} & \sin\theta_{\tilde{f}} \\ -\sin\theta_{\tilde{f}} & \cos\theta_{\tilde{f}} \end{pmatrix}, \qquad (63)$$

with $\theta_{\tilde{f}}$ is the slepton mixing angle.

The four component vectors for up-squarks and down-squarks are, respectively, $(\tilde{u}_{1L},\, \tilde{u}_{2L},\, \tilde{u}_{1R},\, \tilde{u}_{2R})$ and $(\tilde{d}_{1L},\, \tilde{d}_{2L},\, \tilde{d}_{1R},\, \tilde{d}_{2R})$. Thus, the squark squared mass matrices are given by

$$\mathcal{M}^2_{\tilde{u},\tilde{d}} = \begin{pmatrix} M^2_{\tilde{L},c\{s\}} & (M^2_{\tilde{U}\{\tilde{D}\}})_{LL} & m_{c\{s\}}\mathcal{A}_{c\{s\}} & (M^2_{\tilde{U}\{\tilde{D}\}})_{LR} \\ (M^2_{\tilde{U}\{\tilde{D}\}})_{LL} & M^2_{\tilde{L}t\{b\}} & (M^2_{\tilde{U}\{\tilde{D}\}})_{RL} & m_{t\{b\}}\mathcal{A}_{t\{b\}} \\ (M^2_{\tilde{U}\{\tilde{D}\}})_{LR} & (M^2_{\tilde{U}\{\tilde{D}\}})_{RL} & M^2_{\tilde{R}c\{s\}} & (M^2_{\tilde{U}\{\tilde{D}\}})_{RR} \\ (M^2_{\tilde{U}\{\tilde{D}\}})_{LR} & m_{t\{b\}}\mathcal{A}_{t\{b\}} & (M^2_{\tilde{U}\{\tilde{D}\}})_{RR} & M^2_{\tilde{R}t\{b\}} \end{pmatrix}. \tag{64}$$

In order to diagonalize $\mathcal{M}^2_{\tilde{u}\{\tilde{d}\}}$, two rotation 4×4 matrices, $R^{(u)}$ and $R^{(d)}$, one for the *up*-squarks and the other for *down*-squarks, are needed. Thus the squark mass eigenstates $(\tilde{q}'_\alpha)$ and the weak squark eigenstates $(\tilde{q}_\alpha)$ are related by

$$\tilde{q}'_\alpha = \sum R^{(q)}_{\alpha\beta}\tilde{q}_\beta\,. \tag{65}$$

One obtains the squark mass eigenvalues and eigenstates after the diagonalization procedure as indicated in Ref. [46].

5.6 Gluinos, exotic quarks and sfermions

For the exotic quarks and gluinos, their masses are the same as presented at [39].

6 Higgs potential

As usual, the scalar Higgs potential is written as

$$V_{3-3-1} = V_{\rm D} + V_{\rm F} + V_{\rm soft} \tag{66}$$

with

$$\begin{aligned} V_{\rm D} &= -\mathcal{L}_D = \frac{1}{2}\left(D^aD^a + DD\right) \\ &= \frac{g'^2}{12}(\bar{\rho}\rho - \bar{\rho}'\rho' - \bar{\chi}\chi + \bar{\chi}'\chi')^2 \end{aligned}$$

$$
\begin{aligned}
&+\frac{g^2}{8}\sum_{i,j}\left(\bar{\rho}_i\lambda^a_{ij}\rho_j+\bar{\chi}_i\lambda^a_{ij}\chi_j-\bar{\rho'}_i\lambda^{*a}_{ij}\rho'_j-\bar{\chi'}_i\lambda^{*a}_{ij}\chi'_j\right)^2,\\
V_{\mathrm{F}}=&-\mathcal{L}_F=\sum_F \bar{F}_\mu F_\mu\\
&=\sum_i\left[\left|\frac{\mu_\rho}{2}\rho'_i\right|^2+\left|\frac{\mu_\chi}{2}\chi'_i\right|^2+\left|\frac{\mu_\rho}{2}\rho_i\right|^2+\left|\frac{\mu_\chi}{2}\chi_i\right|^2\right],
\end{aligned} \tag{67}
$$

$$
V_{\mathrm{soft}}=-\mathcal{L}_{\mathrm{SMT}}=m^2_\rho\bar{\rho}\rho+m^2_\chi\bar{\chi}\chi+m^2_{\rho'}\bar{\rho'}\rho'+m^2_{\chi'}\bar{\chi'}\chi', \tag{68}
$$

where $m^2_\rho, m^2_\chi, m^2_{\rho'}, m^2_{\chi'}$ have the mass dimension.

All the four neutral scalar components $\rho^0, \chi^0, \rho'^0, \chi'^0$ gain non-zero vacuum expectation values. Expansions of the neutral scalars around their VEVs are usually

$$
\begin{aligned}
<\rho>&=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_\rho+H_\rho+iF_\rho\\ 0\end{pmatrix}, \quad <\rho'>=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_{\rho'}+H_{\rho'}+iF_{\rho'}\\ 0\end{pmatrix},\\
<\chi>&=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ 0\\ v_\chi+H_\chi+iF_\chi\end{pmatrix}, \quad <\chi'>=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ 0\\ v_{\chi'}+H_{\chi'}+iF_{\chi'}\end{pmatrix}.
\end{aligned} \tag{69}
$$

Due to the requirement that the potential to reach a minimum at the chosen VEV's, which is equivalent to the condition of absence of the linear terms in fields, we get a system of constraint equations

$$
\begin{aligned}
12m^2_\rho+3\mu^2_\rho+g^2\left(2v^2_\rho-2v'^2_\rho-v^2_\chi+v'^2_\chi\right)+g'^2\left(v^2_\rho-v'^2_\rho-v^2_\chi+v'^2_\chi\right)&=0,\\
12m^2_{\rho'}+3\mu^2_\rho-g^2\left(2v^2_\rho-2v'^2_\rho-v^2_\chi+v'^2_\chi\right)-g'^2\left(v^2_\rho-v'^2_\rho-v^2_\chi+v'^2_\chi\right)&=0,\\
12m^2_\chi+3\mu^2_\chi-g^2\left(v^2_\rho-v'^2_\rho-2v^2_\chi+2v'^2_\chi\right)-g'^2\left(v^2_\rho-v'^2_\rho-v^2_\chi+v'^2_\chi\right)&=0,\\
12m^2_{\chi'}+3\mu^2_\chi+g^2\left(v^2_\rho-v'^2_\rho-2v^2_\chi+2v'^2_\chi\right)+g'^2\left(v^2_\rho-v'^2_\rho-v^2_\chi+v'^2_\chi\right)&=0.
\end{aligned}
$$

Let us consider the Higgs mass spectrum.

6.1 Neutral scalar Higgs

Let us consider the mass spectrum of the neutral scalar Higgs bosons in the model under consideration. The mass Lagrangian of neutral scalar Higgs can be written in the form

$$\mathcal{L}_{\mathrm{H}} = -\frac{1}{2}\left(H_\rho, H_\chi, H_{\rho'}, H_{\chi'}\right)\mathcal{M}_{\mathrm{H}}^2\ \left(H_\rho, H_\chi, H_{\rho'}, H_{\chi'}\right)^T, \tag{70}$$

where

$$\begin{aligned}\mathcal{M}_{\mathrm{H}}^2 &= \frac{1}{3}(2g^2+g'^2)v_{\chi'}^2\begin{pmatrix} t_1^2 & -a\ t_1\tan\alpha & -a\ t_1 t_2 & a\ t_1 \\ -a\ t_1\tan\alpha & \tan^2\alpha & a\ t_2\tan\alpha & -\tan\alpha \\ -a\ t_1 & a\ t_2\tan\alpha & t_2^2 & -a\ t_2 \\ a\ t_1 & -\tan\alpha & -a\ t_2 & 1 \end{pmatrix}, \\ &= \frac{1}{3}(2g^2+g'^2)v_{\chi'}^2\mathcal{M}_{1\mathrm{H}}^2 \end{aligned} \tag{71}$$

with

$$\begin{aligned} a &= \frac{g^2+g'^2}{2g^2+g'^2},\quad (0<a<1), \\ t_1 &= \frac{v_\rho}{v_{\chi'}},\quad t_2=\frac{v_{\rho'}}{v_{\chi'}}\quad (t_1,\ t_2 \ll 1)\quad \text{and} \\ \tan\alpha &= \frac{v_\chi}{v_{\chi'}}. \end{aligned} \tag{72}$$

Because $\det(\mathcal{M}_{1\mathrm{H}}^2) = 0$, we get a zero-eigenvalue. It is convenient to diagonalize the neutral Higgs mass matrices in two stages. First, we find the transformation for original basis, particularly

$$H = C\ H_1 \leftrightarrow \begin{pmatrix} H_\rho \\ H_\chi \\ H_{\rho'} \\ H_{\chi'} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \cos\alpha & 0 & 0 & \sin\alpha \\ 0 & 0 & 1 & 0 \\ \sin\alpha & 0 & 0 & -\cos\alpha \end{pmatrix}\begin{pmatrix} H_{1\rho} \\ H_{1\chi} \\ H_{1\rho'} \\ H_{1\chi'} \end{pmatrix}. \tag{73}$$

In the new basis $(H_{1\rho},\ H_{1\chi},\ H_{1\rho'},\ H_{1\chi'})$, we have

$$M_{2\mathrm{H}}^2 = C^T\mathcal{M}_{1\mathrm{H}}^2 C$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & t_1^2 & -a\ t_1 t_2 & -\frac{a\ t_1}{\cos\alpha} \\ 0 & -a\ t_1 t_2 & t_2^2 & \frac{a\ t_2}{\cos\alpha} \\ 0 & -\frac{a\ t_1}{\cos\alpha} & \frac{a\ t_2}{\cos\alpha} & \frac{1}{\cos\alpha^2} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & M_{3\times 3} \end{pmatrix}. \tag{74}$$

We would like to remind the reader of the energy scale $v_\chi, v_{\chi'} \gg v_\rho, v_{\rho'}$. This limit leads to $\tan\alpha \gg t_1, t_2$ and the matrix M_3 is a hierarchical matrix. Hence, it is very useful to use the method of block diagonalization in order to find the eigenvectors and and eigenvalues of the matrix M_3.

Let us rewrite matrix $\mathcal{M}^2_{\rm 2H}$ in the basis $(H_{1\chi'},\ H_{1\rho'},\ H_{1\chi}\ )$. In this basis, the matrix M_3 is 3×3 matrix which has form as follows:

$$M_{3\times 3} = \begin{pmatrix} \frac{1}{\cos^2\alpha} & \frac{at_2}{\cos\alpha} & -\frac{at_1}{\cos\alpha} \\ \frac{at_2}{\cos\alpha} & t_2^2 & -a\ t_1 t_2 \\ -\frac{at_1}{\cos\alpha} & -a\ t_1 t_2 & t_1^2 \end{pmatrix}. \tag{75}$$

Next we can use an unitary matrix U_1 such as

$$U_1 = \begin{pmatrix} 1 & \frac{at_2}{\cos\alpha} & -\frac{at_1}{\cos\alpha} \\ \frac{at_2}{\cos\alpha} & 1 & 0 \\ -\frac{at_1}{\cos\alpha} & 0 & 1 \end{pmatrix} \tag{76}$$

in order to transform M_3 into the approximately block-diagonal form and also we change the basis of $H_1 = (H_{1\chi},\ H_{1\rho},\ H_{1\rho'}\ )$ into the new basis $H_2 = (H_{2\chi},\ H_{2\rho},\ H_{2\rho'}\ )$. Details are as follows

$$H_2 = U_1^{-1} H_1, \tag{77}$$

$$U_1^\dagger M_3 U_1 \simeq \begin{pmatrix} \frac{1}{\cos^2\alpha} & 0 \\ 0 & M_{2\times 2} \end{pmatrix} \tag{78}$$

with

$$M_{2\times 2} = \begin{pmatrix} t_1^2 - a^2 t_2^2 & (a-1)at_1 t_2 \\ (a-1)at_1 t_2 & t_2^2 - a^2 t_1^2 \end{pmatrix}. \tag{79}$$

The Eq.(78) proves the existence of the eigenvalues with value $\tan^2\alpha+1$. The matrix $M_{2\times 2}$ produces two eigenvalues as follows

$$
\begin{aligned}
m_{3\rho} &= \frac{1}{2}\left((1-a^2)(t_1^2+t_2^2) - \sqrt{(1+a^2)^2(t_1^2-t_2^2)^2 - 4(a-1)^2a^2t_1^2t_2^2}\right), \\
m_{3\rho'} &= \frac{1}{2}\left((1-a^2)(t_1^2+t_2^2) + \sqrt{(1+a^2)^2(t_1^2-t_2^2)^2 - 4(a-1)^2a^2t_1^2t_2^2}\right) \qquad (80)
\end{aligned}
$$

with two eigenstates are, respectively

$$
\begin{aligned}
H_{3\rho} &= c_\zeta H_{2\rho} - s_\zeta H_{2\rho'}, \\
H_{3\rho'} &= s_\zeta H_{2\rho} + c_\zeta H_{2\rho'} \qquad (81)
\end{aligned}
$$

with $s_\zeta \equiv \sin\zeta, c_\zeta \equiv \cos\zeta$ and ζ is determined through $\tan\zeta$, as follows

$$
\tan 2\zeta = \frac{2a(1-a)t_1t_2}{(1+a)(t_1^2-t_2^2)} \qquad (82)
$$

Let us summarize the neutral Higgs mass spectrum. There is one massless Higgs namely χ'_1 and there are three massive states. One heavy Higgs is $(H_{2\chi})$ with mass

$$
m_{H_{2\chi}^2} = \frac{1}{3}(2g^2+g'^2)(1+\tan^2\alpha)v_{\chi'}^2. \qquad (83)
$$

Two remaining Higgs are $H_{3\rho}, H_{3\rho'}$ with masses, respectively

$$
\begin{aligned}
m^2_{H_{3\rho}} &= \frac{1}{3}(2g^2+g'^2)m_{3\rho}v_{\chi'}^2, \\
m^2_{H_{3\rho'}} &= \frac{1}{3}(2g^2+g'^2)m_{3\rho'}v_{\chi'}^2. \qquad (84)
\end{aligned}
$$

6.2 Pseudo-scalar Higgs

The model under consideration contains four massless pseudo-scalar Higgs bosons, namely $F_\rho, F_\chi, F'_\rho, F'_\chi$, and the mass matrix elements in this case are all equals to zero. It means that all pseudoscalars are massless.

6.3 Singly charged Higgs boson

In the basis $(\rho^-, \rho'^-, \chi^-, \chi'^-)$, the mass Lagrangian for singly charged Higgs bosons has the form

$$
\mathcal{L}^{\text{Singly}}_{\text{charged}} = (\rho^-, \rho'^-, \chi^-, \chi'^-)\, \mathcal{M}^{\text{Singly}}_{\text{charged}}\, (\rho^-, \rho'^-, \chi^-, \chi'^-)^T \qquad (85)
$$

with the mass matrix elements are given by

$$\mathcal{M}_{11} = \frac{g^2}{8} v_{\rho'}^2, \;\; \mathcal{M}_{12} = -\frac{g^2}{8} v_\rho v_{\rho'}, \;\; \mathcal{M}_{13} = \mathcal{M}_{14} = \mathcal{M}_{23} = \mathcal{M}_{24} = 0,$$
$$\mathcal{M}_{22} = \frac{g^2}{8} v_\rho^2, \mathcal{M}_{33} = \frac{g^2}{8} v_{\chi'}^2, \mathcal{M}_{44} = \frac{g^2}{8} v_\chi^2, \mathcal{M}_{34} = \frac{g^2}{8} v_\chi v_\chi'. \tag{86}$$

The matrix $\mathcal{M}^{\text{single}}_{\text{charged}}$ produces two massless states, namely

$$H^+_{\rho_1} = \frac{1}{v_\rho^2 + v_{\rho'}^2} \left(v_\rho v_{\rho'} \rho'^+ + v_{\rho'}^2 \rho^+ \right), \tag{87}$$
$$H^+_{\rho_2} = \frac{1}{v_\chi^2 + v_{\chi'}^2} \left(v_\chi v_{\chi'} \chi'^+ + v_{\chi'}^2 \chi^+ \right), \tag{88}$$

and two massive singly charged Higgs bosons

$$H^+_{\rho_3} = \frac{1}{v_\rho^2 + v_{\rho'}^2} \left(-v_\rho v_{\rho'} \rho'^+ + v_\rho^2 \rho^+ \right), \tag{89}$$
$$H^+_{\rho_4} = \frac{1}{v_\chi^2 + v_{\chi'}^2} \left(-v_\chi v_{\chi'} \chi'^+ + v_\chi^2 \chi^+ \right) \tag{90}$$

and their eigenvalues are, respectively

$$m^2_{H^+_{\rho_3}} = \frac{g^2}{8} \left(v_\rho^2 + v_{\rho'}^2 \right) = m_W^2,$$
$$m^2_{H^+_{\rho_4}} = \frac{g^2}{8} \left(v_\chi^2 + v_{\chi'}^2 \right) = m_V^2. \tag{91}$$

The singly charged Higgs bosons part contains two massless states and two massive states. One has mass equal to the mass of the W gauge boson and other one has mass equal to those of the V gauge boson.

6.4 Doubly charged Higgs boson

The model under consideration contains four doubly charged Higgs bosons, namely ρ^{--}, χ^{--}, ρ'^{--}, χ'^{--}. On this basis, we obtain the mass matrix for doubly charged Higgs boson as follows

$$M^2_{H^{--}} = \frac{g^2}{8}\begin{pmatrix} t_2^2+t_3^2-1 & t_1t_3 & -t_1t_2 & -t_1 \\ t_1t_3 & t_1^2-t_2^2+1 & -t_2t_3 & -t_3 \\ -t_1t_2 & -t_2t_3 & t_1^2-t_3^2+1 & t_2 \\ -t_1 & -t_3 & t_2 & -t_1^2+t_2^2+t_3^2 \end{pmatrix}. \qquad (92)$$

The mass matrix in Eq.(92) produces the mass eigenvalues

$$\begin{aligned} m^2_{H_1^{--}} &= 0, \quad m^2_{H_2^{--}} = \frac{g^2}{8}(v^2_{\chi'} - v^2_\chi + v^2_\rho - v^2_{\rho'}), \\ m^2_{H_3^{--}} &= -m^2_{H_2^{--}}, \; m^2_{H_4^{--}} = \frac{g^2}{8}(v^2_{\chi'} + v^2_\chi + v^2_\rho + v^2_{\rho'}) = m^2_{U^{--}} \end{aligned} \qquad (93)$$

and their mass eigenvectors are, respectively

$$\begin{aligned} H_1^{--} &= \frac{1}{\sqrt{1+t_1^2+t_2^2+t_3^2}}\left(-t_1\rho^{--} + t_3\chi^{--} - t_2\rho'^{--} + \chi'^{--}\right), \\ H_2^{--} &= \frac{1}{\sqrt{1+t_2^2}}\left(t_2\chi^{--} + \rho'^{--}\right), \\ H_3^{--} &= \frac{1}{\sqrt{1+t_1^2}}\left(\rho^{--} + t_1\chi'^{--}\right), \\ H_4^{--} &= C_{H_4^{--}}\left(-t_1\rho^{--} - \frac{(1+t_1^2)t_3}{t_2^2+t_3^2}\chi^{--} + \frac{(1+t_1^2)t_2}{t_2^2+t_3^2}\rho'^{--} + \chi'^{--}\right) \end{aligned} \qquad (94)$$

with $C_{H_4^{--}} = \sqrt{\frac{(1+t_1^2)(1+t_1^2+t_2^2+t_3^2)}{t_2^2+t_3^2}}$.

The mass spectrum of the doubly charged Higgs given in Eqs. (93) shows that the model contains one massive particle with mass equal to that of the doubly charged gauge boson U^{--} and at least one tachyon field, one massless field H_1^{--} which is identified to the Goldstone boson. To remove tachyon in the model, we have to include the following condition: $v^2_{\chi'} - v^2_\chi = v^2_\rho - v^2_{\rho'}$. This leads to appear two other massless particles H_2^{--}, H_3^{--} in the doubly charged Higgs spectrum. The presence of these particles maybe effect to the invisible Z bosons decay modes. Let us consider the invisible decay modes of Z^μ into the massless doubly charged Higgs, namely $Z^\mu \to H_{\rho_2^{--}} H_{\rho_2^{++}}$, $Z^\mu \to H_{\rho_3^{--}} H_{\rho_3^{++}}$. Fig. 6 predicts the invisible decay rate of Z^μ into the massless doubly charged Higgs bosons by studying random scan over the parameter space, such as $w = 10^3 \div 10^5$GeV, $t_v = \frac{v'_\rho}{v_\rho} = 0 \div 100$, $t_w = \frac{v'_\chi}{v_\chi} = 0 \div 10$ and $v'_\rho = 246$ GeV. The obtained result predict the contribution of massless doubly charged Higgs into invisible partial width of Z decay modes is very suppressed. It is suitable to limit on Z-decays into unknown new particles width $\Gamma_{new} < 6.3$ MeV at 95% confidence level given in Ref. [47]. If we compare our predicted results with constraint given in Ref. [47], we obtain very hard constraint on the t_w parameter particularly $t_w = 0.65 \div 0.85$. On the other hand, Fig.7 predicts the Z bosons

decay into two doubly charged Higgs decay width by studying random scan over $t_w = 0.68 \div 0.8$, $v'_\rho = 246$ GeV, $w_\chi = 10^3 \div 10^5$ GeV. The fig.7 plays probability to obtain the small invisible Z decay width ($\Gamma_{\text{invisible}} \leq 2$) MeV is large and the probability is almost independent upon parameter t_v. It means that there is no constraint on the t_v parameter in this case.

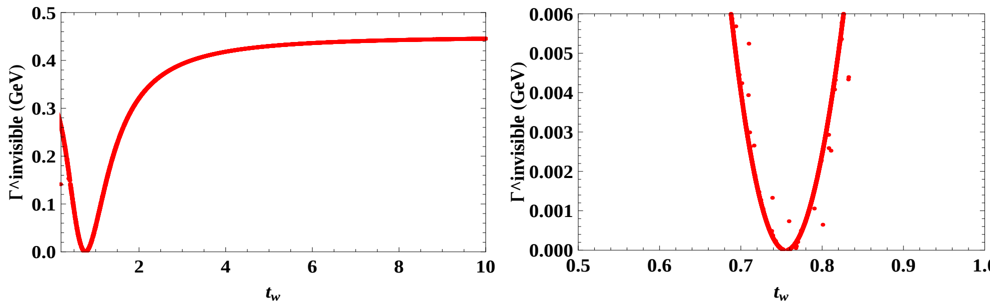

Fig. 6. The invisible decay rate of Z^μ into the massless of the doubly charged Higgs bosons by studying random scan over the parameter space $w = 10^3 \div 10^5$ GeV, $t_w = 0 \div 10$ and $t_v = 0 \div 100$, $v'_\rho = 246$ GeV.

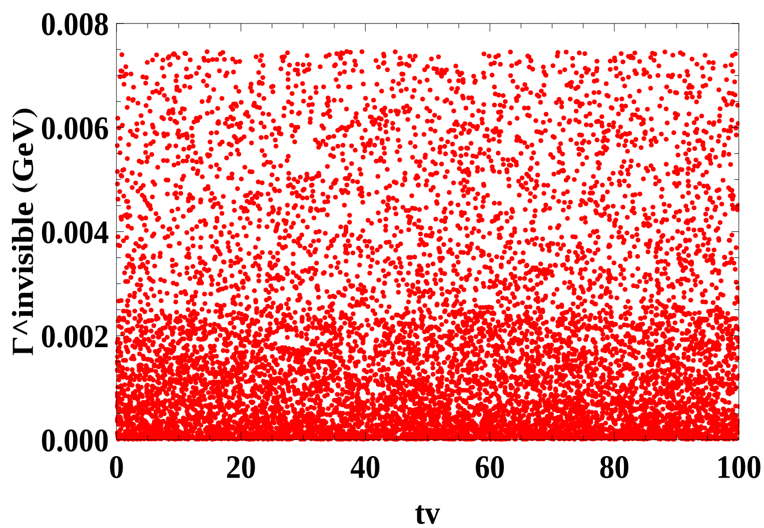

Fig. 7. The invisible decay rate of Z^μ into the massless doubly charged Higgs bosons as the function of t_v by studying random scan over the parameter space $w = 10^3 \div 10^5$ GeV,$t_w = 0.68 \div 0.8$, $v'_\rho = 246$ GeV.

7 Conclusions

We have built the supersymmetric version of the reduced minimal 3-3-1 model with two Higgs triplets. We have studied the mass spectrum of all particles contained in the model. The exact mass spectrum of gauge bosons is studied. In this sector beyond the usual gauge bosons, $W^{\pm}, Z$ gauge bosons, we have two additional charged bosons, $V^{\pm}$ and $U^{\pm\pm}$, and one additional neutral gauge boson Z'. The constraint on the gauge mass is given by $M_{Z'} > M_U > M_V > M_Z > M_W$. In the charged-fermion sector only the tau, top, bottom and charm quarks get their masses at tree level, the others get their masses at one loop level. In the neutrino sector only one neutrino gets mass at tree level, the others two ν_μ and ν_e get their masses at one loop level. The neutrino masses are smaller than those of the charged leptons. It means that we explained the hierarchy of fermion masses in the model under consideration. In the Higgs sector, we can solve exactly the mass eigenstates and mass eigenvalues for charged Higgs bosons. The masses of the massive charged Higgs equal those of the charged gauge bosons, namely $m^2_{H^{\pm}_{\rho 3}} = M^2_{W^{\pm}}, m^2_{H^{\pm}_{\rho 4}} = M^2_{V^{\pm}}$ and $m^2_{H^{--}_4} = M^2_{U^{--}}$. In addition to massive charged Higgs bosons, in the sector of the doubly charged Higgs bosons it also appears the tachyon filed. If the tachyon filed is removed, the model contains two massless doubly charged Higgs bosons. By studying the effect of $Z \to H^{++}_{2,3} H^{--}_{2,3}$ modes on invisible decay width of the Z bosons, we obtain the narrow constraint on $t_w = 0.65 \div 0.8$. In the neutral Higgs bosons, it is very hard to obtain the exact mass spectrum. However, with the help of the relation $u, u' \ll w, w'$, the diagonalization of neutral Higgs boson sector has been performed by using the method of block diagonalization. It leads to the neutral Higgs sector contained three massive states and one massless particle. All pseudo-scalar particles are massless. Some of which is identified to the Goldstones bosons and the remaining pseudo-scalar particles can be identified to the axion. This analysis is not considered in details in this paper.

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A Lagrangian

We are going to write the Lagrangians in terms of the fields in this model

A.1 Lepton Lagrangian

$$\mathcal{L}_{\text{Lepton}} = \int d^4\theta \bar{\hat{L}} \exp\left[2\left(g\frac{\lambda^a}{2}\hat{V}^a\right)\right]\hat{L}$$
$$= \mathcal{L}^{\text{lep}}_{llV} + \mathcal{L}^{\text{lep}}_{\tilde{l}\tilde{l}V} + \mathcal{L}^{\text{lep}}_{l\tilde{l}\tilde{V}} + \mathcal{L}^{\text{lep}}_{\tilde{l}\tilde{l}VV} + \mathcal{L}^{\text{lep}}_{\text{kin}} + \mathcal{L}^{\text{lep}}_{\text{F}} + \mathcal{L}^{\text{lep}}_{\text{D}}, \quad \text{(A.1)}$$

The leptons in this model interact only with the weak $\text{SU}(3)_{\text{L}}$ boson, V^a_μ, and they do not directly couple to the $\text{U}(1)_X$ boson V_μ. The interaction between leptons and gauge bosons in component are given by

$$\mathcal{L}^{\text{lep}}_{llV} = \frac{g}{2}\bar{L}\bar{\sigma}^\mu \lambda^a L V^a_\mu, \quad \text{(A.2)}$$

where λ^a are the usual Gell-Mann matrices.The next part is the slepton gauge boson interaction

$$\mathcal{L}^{\text{lep}}_{\tilde{l}\tilde{l}V} = -\frac{ig}{2}\left[\hat{L}\lambda^a\partial^\mu\bar{\hat{L}} - \bar{\hat{L}}\lambda^a\partial^\mu\hat{L}\right]V^a_\mu. \quad \text{(A.3)}$$

The interaction between lepton-slepton-gaugino is given by the following term

$$\mathcal{L}^{\text{lep}}_{l\tilde{l}\tilde{V}} = -\frac{ig}{\sqrt{2}}(\bar{L}\lambda^a\hat{L}\bar{\lambda}^a_A - \bar{\hat{L}}\lambda^a L\lambda^a_A), \quad \text{(A.4)}$$

and the four interaction between sleptons and gauge bosons

$$\mathcal{L}^{\text{lep}}_{\tilde{l}\tilde{l}VV} = \frac{g^2}{4}V^a_\mu V^{b\mu}\bar{\hat{L}}\lambda^a\lambda^b\hat{L}. \quad \text{(A.5)}$$

The kinetic parts of the leptons and sleptons are

$$\mathcal{L}^{\text{lep}}_{\text{kin}} = -|\partial_\mu\hat{L}|^2 - iL\sigma^\mu\partial_\mu\bar{L}. \quad \text{(A.6)}$$

The last two terms in Eq.(A.1) are the usual F and D terms given by

$$\mathcal{L}^{\text{lep}}_{\text{F}} = |F_L|^2,$$
$$\mathcal{L}^{\text{lep}}_{\text{D}} = \bar{\hat{L}}\lambda^a\hat{L}D^a. \quad \text{(A.7)}$$

A.2 Quark Lagrangian

$$\mathcal{L}_{\text{Quarks}} = \int d^4\theta \;\Big[\bar{\hat{Q}}_1 e^{2[g_s\hat{V}_c + g\hat{V} + (2g'/3)\hat{V}']}\hat{Q}_1 + \bar{\hat{Q}}_\alpha e^{2[g_s\hat{V}_c + g\bar{\hat{V}} - (g'/3)\hat{V}']}\hat{Q}_\alpha$$

$$
\begin{aligned}
&+\ \bar{\hat{u}}_i e^{2[g_s\hat{V}_c-(2g'/3)\hat{V}']}\hat{u}_i+\bar{\hat{d}}_i e^{2[g_s\hat{V}_c+(g'/3)\hat{V}']}\hat{d}_i \\
&+\ \bar{\hat{J}} e^{2[g_s\hat{V}_c-(5g'/3)\hat{V}']}\hat{J}+\bar{\hat{j}}_i e^{2[g_s\hat{V}_c+(4g'/3)\hat{V}']}\hat{j}_i\Big] \\
&=\mathcal{L}_{qqV}+\mathcal{L}_{\hat{q}\hat{q}V}+\mathcal{L}_{q\hat{q}\hat{V}}+\mathcal{L}_{\hat{q}\hat{q}VV}+\mathcal{L}^{\text{quark}}_{\text{cin}}+\mathcal{L}^{\text{quark}}_{\text{F}}+\mathcal{L}^{\text{quark}}_{\text{D}}.
\end{aligned}
\tag{A.8}
$$

as in the lepton sector we can write

$$
\begin{aligned}
\mathcal{L}^{\text{quark}}_{\text{cin}} &= \hat{Q}_i\Box\hat{Q}^*_i+\hat{u}^c_i\Box\hat{u}^{c*}_i+\hat{d}^c_i\Box\hat{d}^{c*}_i+\hat{J}^c_i\Box\hat{J}^{c*}_i-iQ_i\sigma^\mu\partial_\mu\bar{Q}_i-iu^c_i\sigma^\mu\partial_\mu\bar{u}^c_i \\
&- id^c_i\sigma^\mu\partial_\mu\bar{d}^c_i-iJ^c_i\sigma^\mu\partial_\mu\bar{J}^c_i, \\
\mathcal{L}^{\text{quark}}_{\text{F}} &= |F_{Q_i}|^2+|F_{u_i}|^2+|F_{d_i}|^2+|F_{J_i}|^2, \\
\mathcal{L}^{\text{quark}}_{\text{D}} &= \frac{g_s}{2}(\bar{\hat{Q}}_i\lambda^a\hat{Q}_i-\bar{\hat{u}}^c_i\lambda^{*a}\hat{u}^c_i-\bar{\hat{d}}^c_i\lambda^{*a}\hat{d}^c_i-\bar{\hat{J}}^c_i\lambda^{*a}\hat{J}^c_i)D^a_c \\
&+\frac{g}{2}\left(\bar{\hat{Q}}_3\lambda^a\hat{Q}_3-\bar{\hat{Q}}_\alpha\lambda^{*a}\hat{Q}_\alpha\right)D^a \\
&+\frac{g'}{2\sqrt{6}}\left[\frac{2}{3}\bar{\hat{Q}}_3\hat{Q}_3-\frac{1}{3}\bar{\hat{Q}}_\alpha\hat{Q}_\alpha-\frac{2}{3}\bar{\hat{u}}^c_i\hat{u}^c_i+\frac{1}{3}\bar{\hat{d}}^c_i\hat{d}^c_i-\frac{5}{3}\bar{\hat{J}}^c\hat{J}^c+\frac{4}{3}\bar{\hat{j}}^c_\beta\hat{j}^c_\beta\right]D, \\
\mathcal{L}_{qqV} &= \frac{g_s}{2}(\bar{Q}_i\bar{\sigma}^\mu\lambda^aQ_i-\bar{u}^c_i\bar{\sigma}^\mu\lambda^{*a}u^c_i-\bar{d}^c_i\bar{\sigma}^\mu\lambda^{*a}d^c_i-\bar{J}^c_i\bar{\sigma}^\mu\lambda^{*a}J^c_i)g^a_\mu \\
&+\frac{g}{2}(\bar{Q}_3\bar{\sigma}^\mu\lambda^aQ_3-\bar{Q}_\alpha\bar{\sigma}^\mu\lambda^{*a}Q_\alpha)V^a_\mu \\
&+\frac{g'}{2\sqrt{6}}\left(\frac{2}{3}\bar{Q}_3\bar{\sigma}^\mu Q_3-\frac{1}{3}\bar{Q}_\alpha\bar{\sigma}^\mu Q_\alpha-\frac{2}{3}\bar{u}^c_i\bar{\sigma}^\mu u^c_i\right. \\
&\qquad\left.+\frac{1}{3}\bar{d}^c_i\bar{\sigma}^\mu d^c_i-\frac{5}{3}\bar{J}^c\bar{\sigma}^\mu J^c+\frac{4}{3}\bar{j}^c_\beta\bar{\sigma}^\mu j^c_\beta\right)B_\mu, \\
\mathcal{L}_{\hat{q}\hat{q}V} &= \frac{-ig_s}{2}\left[\left(\hat{Q}_i\lambda^a\partial^\mu\bar{\hat{Q}}_i-\bar{\hat{Q}}_i\lambda^a\partial^\mu\hat{Q}_i-\hat{u}^c_i\lambda^{*a}\partial^\mu\bar{\hat{u}}^c_i+\bar{\hat{u}}^c_i\lambda^{*a}\partial^\mu\hat{u}^c_i\right.\right. \\
&-\left.\left.\hat{d}^c_i\lambda^{*a}\partial^\mu\bar{\hat{d}}^c_i+\bar{\hat{d}}^c_i\lambda^{*a}\partial^\mu\hat{d}^c_i-\hat{J}^c_i\lambda^{*a}\partial^\mu\bar{\hat{J}}^c_i+\bar{\hat{J}}^c_i\lambda^{*a}\partial^\mu\hat{J}^c_i\right)g^a_\mu\right] \\
&-\frac{ig}{2}\left(\hat{Q}_3\lambda^a\partial^\mu\bar{\hat{Q}}_3-\bar{\hat{Q}}_3\lambda^a\partial^\mu\hat{Q}_3-\hat{Q}_\alpha\lambda^{*a}\partial^\mu\bar{\hat{Q}}_\alpha+\bar{\hat{Q}}_\alpha\lambda^{*a}\partial^\mu\hat{Q}_\alpha\right)V^a_\mu \\
&-\frac{ig'}{2\sqrt{6}}\left[\frac{2}{3}(\hat{Q}_3\partial^\mu\bar{\hat{Q}}_3-\bar{\hat{Q}}_3\partial^\mu\hat{Q}_3)-\frac{1}{3}(\hat{Q}_\alpha\partial^\mu\bar{\hat{Q}}_\alpha-\bar{\hat{Q}}_\alpha\partial^\mu\hat{Q}_\alpha)\right. \\
&\qquad-\frac{2}{3}(\hat{u}^c_i\partial^\mu\bar{\hat{u}}^c_i-\bar{\hat{u}}^c_i\partial^\mu\hat{u}^c_i)+\frac{1}{3}(\hat{d}^c_i\partial^\mu\bar{\hat{d}}^c_i-\bar{\hat{d}}^c_i\partial^\mu\hat{d}^c_i) \\
&\qquad\left.-\frac{5}{3}(\hat{J}^c\partial^\mu\bar{\hat{J}}^c-\bar{\hat{J}}^c\partial^\mu\hat{J}^c)+\frac{4}{3}(\hat{j}^c_\beta\partial^\mu\bar{\hat{j}}^c_\beta-\bar{\hat{j}}^c_\beta\partial^\mu\hat{j}^c_\beta)\right]B_\mu, \\
\mathcal{L}_{q\hat{q}\hat{V}} &= \frac{-ig_s}{\sqrt{2}}\left[(\bar{Q}_i\lambda^a\hat{Q}_i-\bar{u}^c_i\lambda^{*a}\hat{u}^c_i-\bar{d}^c_i\lambda^{*a}\hat{d}^c_i-\bar{J}^c_i\lambda^{*a}\hat{J}^c_i)\bar{\lambda}^a_c\right. \\
&-\left.(\bar{\hat{Q}}_i\lambda^aQ_i-\bar{\hat{u}}^c_i\lambda^{*a}u^c_i-\bar{\hat{d}}^c_i\lambda^{*a}d^c_i-\bar{\hat{J}}^c_i\lambda^{*a}J^c_i)\lambda^a_c\right] \\
&-\frac{ig}{\sqrt{2}}\left[(\bar{Q}_3\lambda^a\hat{Q}_3-\bar{Q}_\alpha\lambda^{*a}\hat{Q}_\alpha)\bar{\lambda}^a_A-(\bar{\hat{Q}}_3\lambda^aQ_3-\bar{\hat{Q}}_\alpha\lambda^{*a}Q_\alpha)\lambda^a_A\right] \\
&-\frac{ig'}{2\sqrt{3}}\left[\left(\frac{2}{3}\bar{Q}_3\hat{Q}_3-\frac{1}{3}\bar{Q}_\alpha\hat{Q}_\alpha-\frac{2}{3}\bar{u}^c_i\hat{u}^c_i+\frac{1}{3}\bar{d}^c_i\hat{d}^c_i-\frac{5}{3}\bar{J}^c\hat{J}^c+\frac{4}{3}\bar{j}^c_\beta\hat{j}^c_\beta\right)\bar{\lambda}_B\right. \\
&-\left.\left(\frac{2}{3}\bar{\hat{Q}}_3Q_3-\frac{1}{3}\bar{\hat{Q}}_\alpha Q_\alpha-\frac{2}{3}\bar{\hat{u}}^c_iu^c_i+\frac{1}{3}\bar{\hat{d}}^c_id^c_i-\frac{5}{3}\bar{\hat{J}}^cJ^c+\frac{4}{3}\bar{\hat{j}}^c_\beta j^c_\beta\right)\lambda_B\right],
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\hat{q}\hat{q}VV} = & \frac{-1}{4}\left[g_s^2(\bar{\hat{Q}}_i\lambda^a\lambda^b\hat{Q}_i + \bar{\hat{u}}_i^c\lambda^{*a}\lambda^{*b}\hat{u}_i^c\right. \\
& \left. + \bar{\hat{d}}_i^c\lambda^{*a}\lambda^{*b}\hat{d}_i^c + \bar{\hat{J}}_i^c\lambda^{*a}\lambda^{*b}\hat{J}_i^c)g_\mu^a g^{b\mu}\right] \\
& - \frac{1}{4}\left[g^2(\bar{\hat{Q}}_3\lambda^a\lambda^b\hat{Q}_3 + \bar{\hat{Q}}_\alpha\lambda^{*a}\lambda^{*b}\hat{Q}_\alpha)\right]V_\mu^a V^{b\mu} \\
& - \frac{1}{2}\left[g_s g(\bar{\hat{Q}}_3\lambda^a\lambda^b\hat{Q}_3 + \bar{\hat{Q}}_\alpha\lambda^a\lambda^{*b}\hat{Q}_\alpha)\right]g_\mu^a V^{b\mu} \\
& - \frac{g_s g'}{2\sqrt{6}}\left[\frac{2}{3}\bar{\hat{Q}}_3\lambda^a\hat{Q}_3 - \frac{1}{3}\bar{\hat{Q}}_\alpha\lambda^a\hat{Q}_\alpha + \frac{2}{3}\bar{\hat{u}}_i^c\lambda^{*a}\hat{u}_i^c\right. \\
& \left. - \frac{1}{3}\bar{\hat{d}}_i^c\lambda^{*a}\hat{d}_i^c + \frac{5}{3}\bar{\hat{J}}^c\lambda^{*a}\hat{J}^c - \frac{4}{3}\bar{\hat{j}}_\beta^c\lambda^{*a}\hat{j}_\beta^c\right]g^{a\mu}B_\mu \\
& - \frac{gg'}{2\sqrt{6}}\left[\frac{2}{3}\bar{\hat{Q}}_3\lambda^a\hat{Q}_3 + \frac{1}{3}\bar{\hat{Q}}_\alpha\lambda^{*a}\hat{Q}_\alpha\right]V^{a\mu}B_\mu \\
& - \frac{g'^2}{24}\left[\frac{4}{9}(\bar{\hat{Q}}_3\hat{Q}_3 + \bar{\hat{u}}_i^c\hat{u}_i^c) + \frac{1}{9}(\bar{\hat{Q}}_\alpha\hat{Q}_\alpha + \bar{\hat{d}}_i^c\hat{d}_i^c)\right. \\
& \left. + \frac{25}{9}\bar{\hat{J}}^c\hat{J}^c + \frac{16}{9}\bar{\hat{j}}_\beta^c\hat{j}_\beta^c\right]B^\mu B_\mu. \qquad \text{(A.9)}
\end{aligned}
$$

A.3 Scalar Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm Scalar} = & \int d^4\theta\ \left[\ \bar{\hat{\rho}}e^{2g\hat{V}+g'\hat{V}'}\hat{\rho} + \bar{\hat{\chi}}e^{2g\hat{V}-g'\hat{V}'}\hat{\chi}\right. \\
& \left. + \bar{\hat{\rho}}'e^{2g\hat{\bar{V}}-g'\hat{\bar{V}}'}\hat{\rho}' + \bar{\hat{\chi}}'e^{2g\hat{\bar{V}}+g'\hat{\bar{V}}'}\hat{\chi}'\right] \\
= & \mathcal{L}_{\rm F}^{\rm scalar} + \mathcal{L}_{\rm D}^{\rm scalar} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Higgsinos} + \mathcal{L}_{H\hat{H}\hat{V}}, \qquad \text{(A.10)}
\end{aligned}
$$

where the terms with the auxiliary fields can be rewritten as

$$
\begin{aligned}
\mathcal{L}_{\rm F}^{\rm scalar} = & |F_\rho|^2 + |F_\chi|^2 + |F_{\rho'}|^2 + |F_{\chi'}|^2, \\
\mathcal{L}_{\rm D}^{\rm scalar} = & \frac{g}{2}\left[\bar{\rho}\lambda^a\rho + \bar{\chi}\lambda^a\chi - \bar{\rho}'\lambda^{*a}\rho' - \bar{\chi}'\lambda^{*a}\chi'\right]D^a \\
& + \frac{g'}{2\sqrt{6}}\left[\bar{\rho}\rho - \bar{\chi}\chi - \bar{\rho}'\rho' + \bar{\chi}'\chi'\right]D, \qquad \text{(A.11)}
\end{aligned}
$$

while the kinetics terms are

$$
\begin{aligned}
\mathcal{L}_{\rm Higgs} = & (\mathcal{D}_\mu\rho)^\dagger(\mathcal{D}^\mu\rho) + (\mathcal{D}_\mu\chi)^\dagger(\mathcal{D}^\mu\chi) \\
& + (\overline{\mathcal{D}_\mu}\rho')^\dagger(\overline{\mathcal{D}^\mu}\rho') + (\overline{\mathcal{D}_\mu}\chi')^\dagger(\overline{\mathcal{D}^\mu}\chi'), \\
\mathcal{L}_{\rm Higgsinos} = & i\bar{\hat{\rho}}\bar{\sigma}^\mu\mathcal{D}_\mu\hat{\rho} + i\bar{\hat{\chi}}\bar{\sigma}^\mu\mathcal{D}_\mu\hat{\chi} + i\bar{\hat{\rho}}'\bar{\sigma}^\mu\overline{\mathcal{D}_\mu}\hat{\rho}' + i\bar{\hat{\chi}}'\bar{\sigma}^\mu\overline{\mathcal{D}_\mu}\hat{\chi}'. \qquad \text{(A.12)}
\end{aligned}
$$

The covariant derivatives are given by

$$
\begin{aligned}
\mathcal{D}_\mu\phi_i &= \partial_\mu\phi_i - ig\left(\vec{V}_\mu.\frac{\vec{\lambda}}{2}\right)_i^j\phi_j - ig' X_{\phi_i} T^9 B_\mu\phi_i, \\
\overline{\mathcal{D}_\mu}\phi_i &= \partial_\mu\phi_i - ig\left(\vec{V}_\mu.\frac{\vec{\bar{\lambda}}}{2}\right)_i^j\phi_j - ig' X_{\phi_i} T^9 B_\mu\phi_i.
\end{aligned} \tag{A.13}
$$

The interaction between the scalar-gaugino-higgsino is given by

$$
\begin{aligned}
\mathcal{L}_{H\hat{H}\hat{V}} = &-\frac{ig}{\sqrt{2}}\left[\bar{\hat{\rho}}\lambda^a\rho\bar{\lambda}^a_A - \bar{\rho}\lambda^a\hat{\rho}\lambda^a_A + \bar{\hat{\chi}}\lambda^a\chi\bar{\lambda}^a_A - \bar{\chi}\lambda^a\hat{\chi}\lambda^a_A\right. \\
&\left. - \bar{\hat{\rho}}'\lambda^{*a}\rho'\bar{\lambda}^a_A + \bar{\rho}'\lambda^{*a}\hat{\rho}'\lambda^a_A - \bar{\hat{\chi}}'\lambda^{*a}\chi'\bar{\lambda}^a_A + \bar{\chi}'\lambda^{*a}\hat{\chi}'\lambda^a_A\right] \\
&- \frac{ig'}{2\sqrt{3}}\left[\bar{\hat{\rho}}\rho\bar{\lambda}_B - \bar{\rho}\hat{\rho}\lambda_B - \bar{\hat{\chi}}\chi\bar{\lambda}_B + \bar{\chi}\hat{\chi}\lambda_B - \bar{\hat{\rho}}'\rho'\bar{\lambda}_B + \bar{\rho}'\hat{\rho}'\lambda_B\right. \\
&\left. + \bar{\hat{\chi}}'\chi'\bar{\lambda}_B - \bar{\chi}'\hat{\chi}'\lambda_B\right],
\end{aligned} \tag{A.14}
$$

A.4 Gauge Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\text{Gauge}} &= \frac{1}{4}\left[\int d^2\theta\ \left(W^a_c W^a_c + W^a W^a + W' W'\right)\right. \\
&\qquad \left. + \int d^2\bar{\theta}\ \left(\bar{W}^a_c \bar{W}^a_c + \bar{W}^a \bar{W}^a + \bar{W}'\bar{W}'\right)\right] \\
&= \mathcal{L}_{\text{dc}} + \mathcal{L}_D^{\text{gauge}}.
\end{aligned} \tag{A.15}
$$

The kinetic term has the following form

$$
\begin{aligned}
\mathcal{L}_{\text{dc}} = &-\frac{1}{4}\left(G^{a\mu\nu}G^a_{\mu\nu} + W^{a\mu\nu}W^a_{\mu\nu} + B^{\mu\nu}B_{\mu\nu}\right) \\
&- i\left(\bar{\lambda}^a_C\bar{\sigma}^\mu\mathcal{D}^C_\mu\lambda^a_C + \bar{\lambda}^a_A\bar{\sigma}^\mu\mathcal{D}^L_\mu\lambda^a_A + \bar{\lambda}_B\bar{\sigma}^\mu\partial_\mu\lambda_B\right) \quad ,
\end{aligned} \tag{A.16}
$$

with

$$
\begin{aligned}
G^a_{\mu\nu} &= \partial_\mu g^a_\nu - \partial_\nu g^a_\mu - g f^{abc} g^b_\mu g^c_\nu, \\
W^a_{\mu\nu} &= \partial_\mu V^a_\nu - \partial_\nu V^a_\mu - g f^{abc} V^b_\mu V^c_\nu, \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\
\mathcal{D}^C_\mu\lambda^a_C &= \partial_\mu\lambda^a_C - g_s f^{abc} g^b_\mu\lambda^c_C, \\
\mathcal{D}^L_\mu\lambda^a_A &= \partial_\mu\lambda^a_A - g f^{abc} V^b_\mu\lambda^c_A,
\end{aligned} \tag{A.17}
$$

where f^{abc} are the structure constants of the gauge group SU(3), and we have the usual self-interactions (cubic and quartic) of the gauge bosons. The last term in Eq.(A.15) is

$$\mathcal{L}_{\mathrm{D}}^{\mathrm{gauge}} = \frac{1}{2}\left(D^a_C D^a_C + D^a D^a + DD\right). \tag{A.18}$$

A.5 Superpotential

The superpotential of the model is given in Eq.(23). The superpotential in terms of the fields are given by

$$\begin{aligned} W_2 &= \mathcal{L}_F^{W2} + \mathcal{L}_{\hat{\eta}L} + \mathcal{L}_{\mathrm{HMT}}, \\ W_3 &= \mathcal{L}_F^{W3} + \mathcal{L}_{ll\hat{l}} + \mathcal{L}_{llH} + \mathcal{L}_{l\hat{l}\hat{H}} + \mathcal{L}_{l\hat{H}H} \\ &\quad + \mathcal{L}_{\hat{l}HH} + \mathcal{L}_{qqH} + \mathcal{L}_{q\hat{q}\hat{H}} + \mathcal{L}_{lq\hat{q}} + \mathcal{L}_{\hat{l}q\hat{q}}, \end{aligned} \tag{A.19}$$

where

$$\begin{aligned} \mathcal{L}_F^{W2} &= \frac{\mu_\rho}{2}(\rho F_{\rho'} + \rho' F_\rho) + \frac{\mu_\chi}{2}(\chi F_{\chi'} + \chi' F_\chi), \\ \mathcal{L}_{\mathrm{HMT}} &= -\frac{\mu_\rho}{2}\hat{\rho}_i\hat{\rho}'_i - \frac{\mu_\chi}{2}\hat{\chi}_i\hat{\chi}'_i, \\ \mathcal{L}_F^{W_3} &= \frac{1}{3}[3\lambda_1\epsilon F_L\hat{L}\hat{L} + \lambda_2\epsilon(F_L\chi\rho + \hat{L}F_\chi\rho + \hat{L}\chi F_\rho) \\ &\quad + \kappa_1(F_{Q_1}\rho'\hat{d}^c_i + \hat{Q}_1 F_{\rho'}\hat{d}^c_i + \hat{Q}_1\rho' F_{d_i}) \\ &\quad + \kappa_2(F_{Q_1}\chi'\hat{J}^c + \hat{Q}_1 F_{\chi'}\hat{J}^c + \hat{Q}_1\chi' F_J) \\ &\quad + \kappa_3(F_{Q_\alpha}\rho\hat{u}^c_i + \hat{Q}_\alpha F_\rho\hat{u}^c_i + \hat{Q}_\alpha\rho F_{u_i}) \\ &\quad + \kappa_4(F_{Q_\alpha}\chi\hat{j}^c_\beta + \hat{Q}_\alpha F_\chi\hat{j}^c_\beta + \hat{Q}_\alpha\chi F_{j_\beta}) \\ &\quad + \kappa_5(F_{Q_\alpha}\hat{L}\hat{d}^c_i + \hat{Q}_\alpha F_L\hat{d}^c_i + \hat{Q}_\alpha\hat{L}F_{d_i})], \\ \mathcal{L}_{ll\hat{l}} &= -\frac{\lambda_1}{3}\epsilon(LL\hat{L} + \hat{L}LL + L\hat{L}L), \\ \mathcal{L}_{l\hat{H}H} &= -\frac{\lambda_2}{3}\epsilon(L\hat{\chi}\rho + L\chi\hat{\rho}), \\ \mathcal{L}_{qqH} &= -\frac{1}{3}[\kappa_1 Q_1\rho' d^c_i + \kappa_2 Q_1\chi' J^c + \kappa_3 Q_\alpha\rho u^c_i + \kappa_4 Q_\alpha\chi j^c_\beta], \\ \mathcal{L}_{q\hat{q}\hat{H}} &= -\frac{1}{3}[\kappa_1(Q_1\hat{d}^c_i + \hat{Q}_1 d^c_i)\hat{\rho}' + \kappa_2(Q_1\hat{J}^c + \hat{Q}_1 J^c)\hat{\chi}' \\ &\quad + \kappa_3(Q_\alpha\hat{u}^c_i + \hat{Q}_\alpha u^c_i)\hat{\rho} + \kappa_4(Q_\alpha\hat{j}^c_\beta + \hat{Q}_\alpha j^c_\beta)\hat{\chi}], \\ \mathcal{L}_{lq\hat{q}} &= -\frac{\kappa_5}{3}(Q_\alpha\hat{d}^c_i + \hat{Q}_\alpha d^c_i)L, \\ \mathcal{L}_{\hat{l}qq} &= -\frac{\kappa_5}{3}Q_\alpha\hat{L}d^c_i, \end{aligned}$$

$$\mathcal{L}_{\hat{l}HH} = -\frac{\lambda_2}{3}\hat{L}\chi\rho. \qquad \text{(A.20)}$$

References

[1] M. Singer, J. W. F. Valle, and J. Schechter, *Phys. Rev.* **D 22**, 738, (1980).

[2] F. Pisano and V. Pleitez, *Phys. Rev.* **D 46**, 410, (1992); R. Foot, O. F. Hernandez, F. Pisano and V. Pleitez, *Phys. Rev.* **D 47**, 4158, (1993); P. H. Frampton, Phys. Rev. Lett. **69**, 2889, (1992); D. Ng, Phys. Rev. D **49**, 4805, (1994).

[3] V. Pleitez and M. D. Tonasse, *Phys. Rev.* **D 48**, 2353, (1993).

[4] R. Foot, H. N. Long and Tuan A. Tran, *Phys. Rev.* **D 50**, 34, (1994); J. C. Montero, F. Pisano and V. Pleitez, *Phys. Rev.* **D 47**, 2918, (1993); H. N. Long, *Phys. Rev.* **D 54**, 4691, (1996); H. N. Long, *Phys. Rev.* **D 53**, 437, (1996).

[5] S.J. L. Rosner, *Am. J. Phys.* **71**, 302, (2003); A. S. Kronfeld and C. Quigg, *Am. J. Phys.* **78**, 1081, (2010).

[6] P. B. Pal, Phys. Rev. D **52**, 1659, (1995).

[7] J. C. Montero, V. Pleitez and O. Ravinez, Phys. Rev. D **60**, 076003, (1999).

[8] J. C. Montero, C. C. Nishi, V. Pleitez, O. Ravinez and M. C. Rodriguez, Phys. Rev. D **73**, 016003, (2006).

[9] P. V. Dong and H. N. Long, *Int. J. Mod. Phys.* **A 21**,2006, 6677.

[10] B. Meirose and A. A. Nepomuceno, arXiv:1105.6299 [hep-ph].

[11] T. V. Duong and E. Ma, Phys. Lett. **B316**, 307, (1993).

[12] H. N. Long and V. T. Van, *J. Phys.* **G25**, 2319, (1999).

[13] D. Fregolente and M. D. Tonasse, *Phys. Lett.* **B 555**, 7, (2003); H. N. Long and N. Q. Lan, *Europhys. Lett.* **64**, 571, (2003); S. Filippi, W. A. Ponce and L. A. Sanchez, *Europhys. Lett.* **73**, 142 (2006); H. N. Long *et al*, Chapter 6 in *Search for the Higgs Boson.* Nova Science Publishers, New York (2006).

[14] J. C. Montero, V. Pleitez and M. C. Rodriguez, *Phys. Rev.* **D 58**, 1998, 094026.

[15] J. C. Montero, V. Pleitez and M. C. Rodriguez, *AIP Conf. Proc.* **490**, 1999, 397.

[16] J. C. Montero, V. Pleitez and M. C. Rodriguez, *Int. J. Mod. Phys.* **A 16**, 2001, 1147.

[17] J. C. Montero, V. Pleitez and M. C. Rodriguez, *Phys. Rev.* **D 58**, 1998, 097505.

[18] E. Ramirez Barreto, Y. D. A. Coutinho and J. Sa Borges, *Eur. Phys. J.* **C 50**, 2007, 909.

[19] E. Ramirez Barreto, Y. D. A. Coutinho and J. Sa Borges, *Phys. Lett.* **B 632**, 2006, 675.

[20] Y. D. A. Coutinho, P. P. Queiroz Filho and M. D. Tonasse, *Phys. Rev.* **D 60**, 1999, 115001.

[21] W. A. Ponce, Y. Giraldo and L. A. Sanchez, Phys. Rev. D **67**, 075001 (2003)

[22] P. V. Dong, H. N. Long, D. T. Nhung and D. V. Soa, Phys. Rev. D **73**, 035004 (2006); P. V. Dong, D. T. Huong, Tr. T. Huong and H. N. Long, Phys. Rev. D **74**, 053003 (2006); P. V. Dong, H. T. Hung and H. N. Long, Phys. Rev. D **86**, 033002 (2012), for a review, see: P. V. Dong and H. N. Long, Adv. High Energy Phys. **2008**, 739492 (2008), [arXiv:0804.3239(hep-ph)].

[23] J. G. Ferreira, Jr, P. R. D. Pinheiro, C. A. de S. Pires and P. S. Rodrigues da Silva, Phys. Rev. D **84**, 095019 (2011); V. T. N. Huyen, T. T. Lam, H. N. Long, V. Q. Phong, *Neutral currents in reduced minimal 3-3-1 model*, [arXiv:1210.5833(hep-ph)] to be published

[24] H. N. Long and P. B. Pal, Mod. Phys. Lett. **A13**, 2355, (1998).

[25] J. C. Montero, V. Pleitez and M. C. Rodriguez, *Phys. Rev.* **D65**, 035006 (2002).

[26] M. Capdequi-Peyranère and M. C. Rodriguez, *Phys. Rev.* **D65**, 035001 (2002).

[27] J. C. Montero, V. Pleitez and M. C. Rodriguez, *Phys. Rev.* **D 70**, 075004 (2004).

[28] D. T. Huong, M. C. Rodriguez and H. N. Long, arXiv:hep-ph/0508045.

[29] P. V. Dong, D. T. Huong, M. C. Rodriguez and H. N. Long, *Eur. Phys. J.* **C48**, 229 (2006).

[30] P. V. Dong, D. T. Huong, M. C. Rodriguez and H. N. Long, *Nucl. Phys.* **B772**, 150 (2007).

[31] S. Sen, *Phys. Rev.* **D 76**, 115020 (2007).

[32] S. Sen and A. Dixit, *Phys. Rev.* **D71**, 035009 (2005).

[33] R. A. Diaz, R. Martinez and F. Ochoa, *Phys. Rev.* **D72**, 035018, (2005).

[34] L. A. Sanchez, W. A. Ponce and J. M. Mira, *Eur. Phys. J.* **C42**, 205 (2005).

[35] J. M. Mira, W. A. Ponce, D. A. Restrepo and L. A. Sanchez, *Phys. Rev.***D67**, 075002 (2003).

[36] J. C. Montero, V. Pleitez and M. C. Rodriguez, *Phys. Rev.* **D 65**, 095008 (2002).

[37] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, 2nd edition, Princeton University Press, Princeton NJ, (1992).

[38] H. E. Haber and G. L. Kane, Phys. Rep. **117**, 75 (1985).

[39] M. C. Rodriguez, Journ. Mod. Phys. **2**, 1193 (2011).

[40] L. Girardello and M. T. Grisaru, *Nucl. Phys.* **B194**, 65 (1982).

[41] C. M. Maekawa and M. C. Rodriguez, JHEP **0604**, 031 (2006).

[42] B. C. Allanach *et al*, *Eur.Phys.J.***C25**, 113 (2002).

[43] N. Ghodbane and Hans-Ulrich Martyn, hep-ph/0201233.

[44] http://spa.desy.de/spa/

[45] C. M. Maekawa and M. C. Rodriguez, JHEP **0801**, 072 (2008).

[46] A. M. Curiel, M. J. Herrero and D. Temes, Phys.Rev. **D**67 (2003) 075008..

[47] Mönig, K, DELPHI 97-174 PHYS 748; K Nakamura (Particle Data Group) J. Phys. G: Nucl. Part. Phys. **37** 075021 (2010).